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Asset Allocation and the Business Cycle

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Asset Allocation and the Business Cycle

– Daniel Maximilian Fehrle –

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List of Acronyms

BCA Business Cycle Accounting

DSGE Dynamic Stochastic General Equilibrium

ECB European Central Bank

EIS elasticity of intertemporal substitution

GDP Gross Domestic Product

IRF impulse response function

JKKST Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019)

JST Jordà, Schularick, and Taylor (2019)

LLV likelihood value

MLE maximum-likelihood estimation

MSE mean squared error

PCE Private Consumption Expenditure

RBC Real Business Cycle

RRA relative risk aversion

VAR vector autoregression

VAT value-added tax

List of Publications and the Author's Contribution

The thesis includes the following three essays:

FEHRLE, D. (2019): "Housing and the business cycle revisited," *Journal of Economic Dynamics and Control*, 99, 103–115.

FEHRLE, D., AND C. HEIBERGER (2020): "The return on everything and the business cycle in production economies" .

FEHRLE, D., AND J. HUBER (2020): "Business cycle accounting for the German fiscal stimulus program during the Great Recession" .

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Chapter 1

Introduction

During the Great Recession more than half of the total decline in Gross Domestic Product (GDP) in the US was, according to Berger and Vavra (2015), assigned to decreases in residential investment and new durable consumption goods. Further, residential investment had accounted for 58 percent and new durable consumption goods for 26.6 percent of GDP changes during recessions between 1960 and 2013. Since both are highly volatile, pro-cyclical, and weaken GDP in particular at the beginning of a recession, Leamer (2007) concludes, "Housing IS the business cycle". Residential structures and durable consumption goods are similar by nature; in contrast to most real assets, both are not input factors of production, but increase the consumer's utility directly. As a consequence, they are rather held by final consumers than firms, and thus, such assets are more private and serve a specific purpose. The similar internal and business cycle characteristics motivate the topics of the three essays in this thesis. These essays deal with puzzles and solution approaches or applications in differentiating between productive, market and utility augmenting, private assets inside Dynamic Stochastic General Equilibrium (DSGE) models. DSGE models are a workhorse of modern business cycle research.

Although DSGE models are well-established in macroeconomics, they are also often subject to criticism. The thesis does not contribute to this debate, I rather want to make a point that each element of this acronym is necessary and offers further advantages to study the object under investigation. First, the large contribution of private investment goods to the business cycle raises questions about their effects on the overall economy. Second, the high volatility and the pro-cyclicality raises questions about the effects of overall economic activity onto these investment goods. In general equilibrium all prices and quantities are interdependent, and thus, this is an appropriate framework to study such feedback systems. Since it is indisputable that business cycles evolve over time and that the future is uncertain, it is essential to take dynamics and stochastics into account. A second argument for dynamics is the missing rationale behind investments in a static world. Consequently, in a dynamic and stochastic economy, the decisions on today's investment depend on an uncertain future, and thereby, the agent's current activities are determined to some extent by expectations of the future. The model's stochastic feature enables statistical inference; thus, in this thesis maximum-likelihood and Bayesian estimation or methods of moments are applied wherever meaningful.

As DSGE models are an appropriate tool, the distinction of market and privately used assets has a long tradition inside this framework. Early attempts of such distinctions by Greenwood and Hercowitz (1991) and Benhabib, Rogerson, and Wright (1991) established a strand of literature on heterogeneous capital goods, as their models tend to generate anomalies in investment activities. All models in the thesis at hand tend to generate these anomalies as well, namely excessive volatility and counterfactual negative correlated movements of the investment goods. The intuition is straightforward: From an agent's perspective the quantity of interest is the size of the capital stocks which depreciates annually, say about 10 percent. Consequently, a reduction of 100 percent in the quantity of an investment leads only to a 10 percent reduction of the concerning capital stock per year and an investment has a half-life of approximately 7 years. Thus, short-run investment reductions have a small impact and increases have persistent effects. Different investment goods are therefore highly substitutable in the short-run, and hence, small temporary changes in the relative price – or the usability – lead to excessive adjustments and negatively correlated fluctuations. Shocks change the relative usability of the assets because the specific purpose of the private asset entails an asymmetry. For instance, the market asset can be used to build up the private asset but not vice versa, and to that end, a positive productivity shock increases investments in the market asset first. This asymmetry is reinforced if only market assets are directly related to productivity.

Chang (2000) shows that adjustment costs to investment or capital may solve the excess volatility and negatively correlated movement puzzle. As investment adjustments become costly, substitution becomes costly, too, which limits the effect of relative price changes on the demand side. Based on this result, all models in the three essays have the following common features: at least two types of assets, one augments labor productivity and one utility, and both are subject to adjustment costs. However, the interpretation of the utility-augmenting asset differs. In Chapter 2 and 3 this asset is the stock of houses, which is a composite of residential structures and land, a fixed input factor. In Chapter 4 the utility-augmenting asset is represented by the stock of durable consumption goods.

While the field of housing and the business cycle was a niche, the Great Recession made it fashionable (see e.g. Davis and Nieuwerburgh (2015) or Iacoviello (2010)). The pioneer work of Davis and Heathcote (2005) takes the neoclassical perspective, where markets are perfect and the supply side induces the entire business cycle fluctuation. The pioneer work of Iacoviello (2005) uses the New Keynsian perspective, where markets are imperfect and the demand side accounts for business cycles as well. The former work tries to account for stylized facts of the business cycle by modeling a multi-sectoral input-output linked supply structure. The following stylized facts are valid for the most developed economies: i) residential investment is at least moderately more volatile than business investment, ii) house prices are at least twice as volatile as GDP, and iii) house prices, business investment as well as GDP are positively correlated with residential investment. While Davis and Heathcote (2005) achieve success in explaining the stylized facts concerning the quantities, they fail doing so for the facts concerning house prices. Iacoviello (2005) illustrates an amplification of demand shocks through housing collateral constraints, which coincides with observations made during the Great Recession. This is why a huge strand of literature builds on Iacoviello (2005) and reduces the structural form of the supply side to the extent that some business cycle statistics could barely be matched. Further, the literature concludes that supply-side – or strictly spoken technology – shocks solely cannot account for the positive correlation of prices and quantities in the housing market and the high residential investment volatility at once (see e.g. Iacoviello and Neri (2010)). This is first due to the inability of the Davis and Heathcote (2005) model to account for the positive correlation between house prices and residential investment. Additionally, in partial equilibrium, a shift in the demand curve leads to positively correlated prices and quantities and a shift in the supply curve has the opposite effect. However, to account for the volatility of residential investments, it is mostly assumed that large shocks affect the supply of residential goods. Chapters 2 and 3 check to which extent the stylized facts, including quantities and prices, could be explained in neoclassic – strictly spoken Real Business Cycle (RBC) – models with deeper structures and real frictions. To also gain insights into asset pricing in general, Chapter 3 additionally investigates asset return behavior.

In detail, in the first essay (Chapter 2) "Housing and the business cycle revisited", already published as Fehrle (2019), I argue that Davis and Heathcote (2005) strip the supply side down by assuming that sectoral productivity is driven by a reduced-form process with correlated shocks. The success of the model's ability to account for the stylized facts of the quantities depends heavily on this assumption since without correlated shocks in sectoral productivity e.g. the quantities of both investment types are negatively correlated. The assumption of correlated shocks keeps changes in the relative price of business and residential investments small, and thereby, prevents large substitutions but transfers explanation content outside the model towards the reduced-form process.

With this result in mind, I show that variable capital utilization, business investment adjustment costs, and a higher cost share of land in new houses resolve the negative correlation puzzle without relying on correlated shocks. Note that land acts similarly to adjustment costs. The introduction and the increase of adjustment costs, respectively, limit the substitution effect to the extent that the income effect dominates. Variable capital utilization enhances the income effect. To this end, all GDP subaggregates are positively correlated and the model indeed keeps the explanatory power of the concerning volatilities well. The income effect of productivity shocks on inputs which are less intensive in the production of residential investment goods is so prevailing that house prices and residential investment are positive correlated as well. From a supply and demand model perspective, the mentioned shocks hardly affect the supply side of new houses because they are secondary in their production, however, as the income effect dominates they increase the demand for a given price. These demand curve shifts result in a volatility of house prices which is twice as high as in the Davis and Heathcote (2005) benchmark. Nevertheless, house prices are still less volatile than GDP. Lastly, the nature of the modeled business investment adjustment costs penalize rapid changes between current and past investment. This does not apply due the effect of land on new houses, and for this reason, business investment lags residential investment in accordance with the data.

The second essay "The return on everything and the business cycle in production economies" (Chapter 3), which is joint work with Christopher Heiberger and the current version of the working paper by Fehrle and Heiberger (2020), checks the ability of RBC models to account additionally for stylized facts which characterize asset returns based on the database from Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019). The motivation for this is two-sided. On the one hand, stochastic growth models, in particular RBC models, typically fail to reproduce the empirically observed asset return characteristics (see e.g. Mehra and Prescott (1985) and Weil (1989)). Thus, most of the RBC models which focus on housing do not consider asset return implications. On the other hand, there is a strand of literature trying to solve asset return puzzles inside the RBC framework. However, traditionally the approaches take only equity into account, although housing accounts for 50 percent of an advanced economy's total wealth. When housing is taken into account, additional puzzles emerge, i.e. housing risk premia are only moderately lower than equity risk premia, but the return on housing is far less volatile.

First, we show that a trade-off occurs between accounting for business cycle statistics and for sizable risk premia when we include housing in approaches in the manner of Jermann (1998) and Boldrin, Christiano, and Fisher (2001). These approaches rely on habit formation, adjustment costs, and partly on limited sectoral mobility. The key figure is the transformability of consumer goods into new homes and vice versa. As long as they are easily convertible, the household can smooth his consumption bundle over the cycle; in recessions by transforming residential investment goods into consumption goods and vice versa in expansions. That is because the stock of houses depreciates slowly and the elasticity of housing in utility is low. By the same token, consumption depreciates by definition with 100 percent and its elasticity in utility is high. This insurance option increases the willingness to take over aggregated risk to the extent that risk premia virtually vanish. However, the option involves a pro-cyclical demand effect for new houses, which results in a high volatility of residential investment and house prices as well as a positive correlation between the two. By restricting the transformability, the household charges higher premia increase, but the models fail to account for the business cycle statistics – both because the insurance option is no longer feasible. Hence, those approaches cannot account for business cycle statistics and sizable risk premia at once. Additionally, all models generally fail to account for second moments of asset return rates.

A second approach with time-varying disaster risk and Epstein-Zin preferences is more promising. Since disasters are introduced through large reductions in total factor productivity and large depreciations of productive capital as well as residential structures, the household cannot insure himself against disaster risk even if consumption, business and residential investment goods are homogeneous. Thus, the model accounts for sizable risk premia. A lower elasticity of house prices in comparison to Tobin's 'q' (capital prices) lowers housing premia and raises the volatility of residential investment, both, relative to business investment. The time-varying feature contributes to explanations for the volatility of the return on housing, of the return on aggregated risk and the risk-free rate as well as the volatility of all quantities and house prices relative to GDP. Further, time-varying disaster risk increases the positive correlation between residential investment and house prices. The mechanism of the time-varying disaster risk feature is as follows: An increase in the disaster probability increases the expectations on future stock depreciations and decreases the expectation on future total factor productivity. Thus, the household deinvests. Since the elasticity of house prices is lower, relative changes in residential investment are higher than in business investment. Further, despite total productivity remain, the demand for assets decreases, and consequently, asset prices fall. Hence, asset quantities and prices move correlated in the absence of productivity shocks. Leverage also helps to differentiate between the return on housing and equity and the concerning standard deviations. Nevertheless, the volatility of the return on equity is still too low. This results in the major drawback of the model: it cannot differentiate between the Sharp ratio of housing and equity.

The large contribution to the business cycle of residential investment and new durables, their high volatility, and their pro-cyclicality together in combination with the high substitutability of such investment goods make them an interesting object of stabilizing policy. Note that investment goods have also a high intertemporal substitutability for themselves as long as the depreciation rate is low. Thus, small changes in taxes and subsidies have a leverage effect. Thereby, e.g. by shifting a fraction of investment activities from a potentially recovered future into a recessive present, one can smooth the business cycle with low fiscal spending. Even a dynamic counter-cyclical tax policy would be conceivable. However, caution is recommended. The mechanisms in Chapter 2 and 3 imply that the residential investment behavior could be efficient in the sense that it is first-best policy – given the states of nature. In those economies any additional stabilizing policies would lead to losses in the household's utility. This makes stabilizing policies more complicated than just looking for cost-effective tools which smooth pro-cyclical and high-volatile GDP subaggregates. Having said that, there are also plausible arguments for such fiscal stimuli, which is why the German government included a durables subsidy, i.e. a cash for clunkers program, in their fiscal stimulus program during the Great Recession 2008 and 2009.

The third essay (Chapter 4) "Business cycle accounting for the German fiscal stimulus program during the Great Recession", which is joint work with Johannes Huber and the current version of the working paper by Fehrle and Huber (2020), considers the Great Recession in Germany, the concerning stabilization policy and in particular the cash for clunkers program through the lens of the stochastic neoclassical growth model.

The Business Cycle Accounting (BCA) framework as proposed by Chari, Kehoe, and McGrattan (2007) is based on the benchmark RBC model. Additionally, in nearly every market time-varying distortions take place. These distortions – called wedges – are modeled as taxes, stochastic productivity or government spending. As Chari, Kehoe, and McGrattan (2007) show, these wedges are the reduced form of a broad set of frictions and market imperfections which are mappable onto the wedges. For example a model with sticky wages is equivalent to a model with a labor market wedge. We include wedges to the variables government consumption, durables, investment, labor, net exports, and efficiency and show how to map the measures of the fiscal stimulus program towards them. We further include adjustment costs to the capital stock and the stock of durables. This avoids that the durables and the investment wedge are perfectly correlated to prevent the counterfactual negative correlated movements of the concerning quantities. All wedges are driven by a Markov process, commonly parameterized as a vector autoregression (VAR) with one lag. We measure the wedges and the adjustment costs with frequentist inference. Afterwards, we feed the measured wedges back into the model one by one, to assess how much of movements in GDP, subaggregates and hours worked can be attributed to the particular wedge.

Our findings suggest that the efficiency wedge drove the Great Recession in Germany. The investment wedge and the net exports wedge contributed substantially to the crisis. The government consumption wedge and in particular the durables wedge acted counter-cyclically. We attribute the latter to the cash for clunkers program or more general to durables subsidies. We conclude that this subsidy was more effective than pure government consumption, since the effects were similar but the expenditures for the subsidies were far lower. Previous BCA applications for Germany do not differentiate between productive investment and new durables. As their wedges affect the business cycle in different directions based on a government intervention, they potentially underestimate financial frictions which are mappable towards the investment wedge.

Since the application of BCA is reported to be somewhat difficult and there is no straight methodological implementation of BCA, we describe in addition a well-performing procedure, which we apply in this study. The procedure includes approaches which are scattered in the literature or we developed them.

Bibliography

BENHABIB, J., R. ROGERSON, AND R. WRIGHT (1991): “Homework in Macroeconomics: Household Production and Aggregate Fluctuations,” *Journal of Political Economy*, 99(6), 1166–1187.

BERGER, D., AND J. VAVRA (2015): “Consumption Dynamics During Recessions,” *Econometrica*, 83(1), 101–154.

BOLDRIN, M., L. J. CHRISTIANO, AND J. D. M. FISHER (2001): “Habit Persistence, Asset Returns, and the Business Cycle,” *American Economic Review*, 91(1), 149–166.

CHANG, Y. (2000): “Comovement, excess volatility, and home production,” *Journal of Monetary Economics*, 46(2), 385–396.

CHARI, V. V., P. J. KEHOE, AND E. R. MCGRATTAN (2007): “Business Cycle Accounting,” *Econometrica*, 75(3), 781–836.

DAVIS, M. A., AND J. HEATHCOTE (2005): “Housing and the business cycle,” *International Economic Review*, 46(3), 751–784.

DAVIS, M. A., AND S. V. NIEUWERBURGH (2015): “Housing, Finance, and the Macroeconomy,” in *Handbook of Regional and Urban Economics*, 753–811. Elsevier.

Fehrle, D. (2019): "Housing and the business cycle revisited," *Journal of Economic Dynamics and Control*, 99, 103–115.

Fehrle, D., and C. Heiberger (2020): "The return on everything and the business cycle in production economies" .

Fehrle, D., and J. Huber (2020): "Business cycle accounting for the German fiscal stimulus program during the Great Recession" .

Greenwood, J., and Z. Hercowitz (1991): "The Allocation of Capital and Time over the Business Cycle," *Journal of Political Economy*, 99(6), 1188–1214.

Iacoviello, M. (2005): "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review*, 95(3), 739–764.

——— (2010): *Housing in DSGE Models: Findings and New Directions,* 3–16. Springer Berlin Heidelberg, Berlin, Heidelberg.

Iacoviello, M., and S. Neri (2010): "Housing Market Spillovers: Evidence from an Estimated DSGE Model," *American Economic Journal: Macroeconomics*, 2(2), 125–164.

Jermann, U. J. (1998): "Asset pricing in production economies," *Journal of Monetary Economics*, 41(2), 257–275.

Jordà, Ò., K. Knoll, D. Kuvshinov, M. Schularick, and A. M. Taylor (2019): "The Rate of Return on Everything, 1870–2015," *The Quarterly Journal of Economics*, 134(3), 1225–1298.

Leamer, E. (2007): "Housing IS the Business Cycle," Discussion paper.

Mehra, R., and E. C. Prescott (1985): "The equity premium: A puzzle," *Journal of Monetary Economics*, 15(2), 145–161.

Weil, P. (1989): "The equity premium puzzle and the risk-free rate puzzle," *Journal of Monetary Economics*, 24(3), 401–421.

Chapter 2

Housing and the Business Cycle revisited

— *Daniel Fehrle* —

The essay of Fehrle (2019) is already published as:

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Chapter 3

The return on everything and the business cycle in production economies

— *Daniel Fehrle and Christopher Heiberger*—

3.1 Introduction

The seminal publications of Mehra and Prescott (1985) and Weil (1989) have issued a challenge to macroeconomic models: explaining the historically observed sizeable equity premium (excess of the return on a stock market index over the return of a relatively riskless security) together with the low risk-free rate for a reasonable degree of risk aversion. While standard Real Business Cycle (RBC) models are successful in accounting for important stylized facts of the business cycle, they typically fail to reproduce the empirically observed characteristics of asset returns. Over the past years, different approaches have been suggested by the literature in order to solve the puzzle. To name but a few, Jermann (1998) combines modifications to the standard preference structure with frictions in the adjustment of input factors, Boldrin, Christiano, and Fisher (2001) add frictions in the allocation of input factors, and Gourio (2012) introduces a risk for rare but severe economic disasters. While these models are able to replicate the empirical risk premium on stocks, they commonly ignore an asset which, according to Jordà, Schularick, and Taylor (2019) (JST), forms roughly 50 percent of an advanced economy's total wealth, namely housing. Looking at it the other way round, the RBC literature which focuses on housing generally does not consider implications for asset returns either. In the present paper, we therefore aim to combine the two strands of the literature with the objective to mutually explain key asset pricing and business cycle statistics including housing.

The new database built by Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019) (JKKST) covers long term data on the return on equity, on the return on housing and on the return on total wealth as well as data on government bills and bonds for 16 advanced economies. Based on this new data, JST re-measure the return rates on a representative investor's total portfolio and find that the risk premium puzzle by Mehra and Prescott (1985) in fact further worsens if attention is not restricted to equity only: the Sharpe ratio of housing is even larger than the Sharpe ratio of equity. Their result raises three questions. First, are existing approaches capable of explaining even larger Sharpe ratios than previously required for risky assets, second, can return rates and volatilities of various assets be differentiated, and third, can different Sharpe ratios between the two risky assets be matched. They show that several popular approaches which were previously shown to be successful in reproducing the return rates on equity, turn out less successful once the return on housing and the return on total wealth are also taken into consideration. While the study of JST considers various different approaches including habit formation as in Abel (1990) and Campbell and Cochrane (1999) or disaster risk with and without recursive utility as in Rietz (1988), Barro (2006), and Bansal and Yaron (2004), they focus on endowment economies throughout.

We think that studying more general asset pricing statistics also in production economies is important for various reasons. First, the analysis of multiple assets asks for an explanation of the empirically observed differences in the mean return rates, volatilities and Sharpe ratios. For example, in the Lucas (1978) framework for asset prices, different Sharpe ratios of assets can only be realized if the correlations of the assets' returns with the model's stochastic discount factor differ. While different volatilities of returns and different correlations between return rates and consumption growth are introduced exogenously in endowment economies, the explanation of these features becomes an important exercise in production economies. Second, as argued by Cochrane and Hansen (1992), any friction which can help to reproduce asset pricing statistics may on the other hand have empirically counterfactual implications for business cycle statistics. Hence, the effects of such frictions on both, asset prices and business cycles, should be analyzed simultaneously within the framework of RBC models. Third, many mechanisms which can explain risk premia in endowment economies may fail in general equilibrium since the household can alter his plans to smooth consumption and thereby insure himself. Fourth, the business cycle is potentially the macroeconomic phenomenon with the largest effects on asset returns. Hence, explaining the key facts of asset return rates and the business cycle in the same internally consistent model is important in order to gain insights into this relationship. Fifth, RBC models are the backbone for a broader class of Dynamic Stochastic General Equilibrium (DSGE) models used for stabilization policy analysis. For this purpose, an unsatisfactory performance with regard to asset pricing statistics may constitute a significant shortcoming of these models. For example, high risk premia may diminish investment activities even if the riskless interest rate is low.

In our analysis we simultaneously focus on partly puzzling stylized facts of asset prices and the business cycle. These stylized facts are identified as common features from historic data which are valid for several developed countries over long time periods. Among the stylized facts which characterize asset returns are: i) a stable risk-free rate smaller than 2.25 percent, ii) return rates on equity moderately larger than returns on housing, iii) risk premia on equity, on housing and on total risk larger than 3 percent, iv) return rates and premia on equity which are at least twice as volatile as return rates and premia on housing and on total risk, and v) a Sharpe ratio of housing significantly larger than the Sharpe ratio of equity. Turning to business cycle statistics, they reveal the following important characteristics: i) residential investments are at least moderately more volatile than business investments, ii) house prices are at least twice as volatile as Gross Domestic Product (GDP), and iii) house prices, business investments as well as GDP are positively correlated with residential investments, and the correlation between house prices and GDP is also positive. Jaimovich and Rebelo (2009) designate the ability to generate correlated movements of subaggregates as a litmus test for RBC models.

The starting point of our analysis is a variation of the Jermann (1998) model with exogenous labor but extended by a separate housing stock. Following Davis and Heathcote (2005), the stock of houses differs from productive capital in two aspects. First, it enters the household's utility function whereas productive capital enters the production function and, second, houses depreciate at a lower rate. While we assume the same capital adjustment costs in line with the 'q-theory' for business investments as in Jermann (1998), convex adjustment costs for housing arise from the fact that new houses require that residential structures must be linked to land. Moreover, we first assume that business investments, residential investments and the consumption good are homogenous goods. This assumption together with the fact that the elasticity of housing in the household's consumption bundle as well as the depreciation rate of houses are both small, allows the household to conveniently smooth his consumption bundle across different states of nature through optimal adjustment of residential investments in response to technology shocks. In consequence, risk premia in the Jermann (1998) model with housing vanish even when large habits in consumption and housing are assumed. Nevertheless, the model can predict a pro-cyclical demand effect for residential investments. House prices fluctuate more than GDP and the model reproduces the observed co-movements from the data.[1]

In a second step we restrain the household's option to smooth his consumption bundle after the shock's realization. We consider a two sector model where residential investments are produced in one sector whereas production of business investments and of the consumption good takes place in a second sector. The productive capital stock is sector-specific and immobile, and subject to adjustment costs in both sectors as in Fehrle (2019). This model can be interpreted as a stripped down version of the multi-sectoral model by Davis and Heathcote (2005) and is similar to the model of Nguyen (2018). Sticking at first to the assumption of exogenous labor supply, the model can produce moderate risk premia. The model's ability to explain sizeable risk premia is lost once labor supply is determined endogenously, but can be recovered if labor mobility between the sectors is limited similar to Boldrin, Christiano, and Fisher (2001). However, the model performs worse with respect to the residential business cycle statistics and in particular fails to generate co-moving economic activity between the two sectors. In consequence, we conclude that the model cannot explain sizeable risk premia and the observed co-moving economic activity simultaneously. Further, return rates turn out far too volatile in the model. The standard deviation of the risk-free rate exceeds its empirical counterpart by a factor of 8 while the return rates on housing and on the total portfolio are more than 4 and more than 2 times, respectively, as volatile as in the data. Moreover, the model cannot explain any of the empirically observed differences between equity and housing.

Including housing into disaster economies turns out more promising. We consider an otherwise standard RBC model with housing where economic disasters are introduced through large negative shocks which reduce total factor productivity and also destroy productive capital and residential structures to the same extent. Moreover, the model features time-varying disaster risk and recursive preferences of the class introduced by Epstein and Zin (1989). Different elasticities of Tobin's q and of house prices help to explain differences in the mean and in the volatilities between returns on unlevered equity and on housing while leverage additionally helps to differentiate the effect. Keeping the coefficient of relative risk aversion to a moderate level of 5.5, the model can explain a low return rate on government bonds of 1.31 percent on average (1.57 percent in the US data) and is able to replicate an equity premium of 6.56 percent (5.88 percent in the US data). In accordance with the data, return rates on housing turn out moderately lower than on equity and the housing premium in the model is 3.00 percent (compared to 4.45 percent in the US data). The total risk premium in the model turns out to be 4.98 percent and closely matches the value from the data (5.27 percent in the US data). Next to mean return rates and premia, the model can also match the low volatility of government bonds fairly well. Time-varying disaster risk helps to increase the volatility of the risky assets' returns and allows to closely reproduce the standard deviations of returns and premia on housing as well as on total risk. However, the standard deviations of returns and premia on equity remain too small. The model can closely replicate the Sharpe ratio of housing and the Sharpe ratio of the total portfolio from the data but does not match the significantly smaller Sharpe ratio of equity. Although the premia and their volatilities differ between the two risky assets, the model cannot generate different Sharpe ratios.

The disaster model is able to generate relative volatilities of business investments, residential investments and house prices which all fit the data. Business investments are almost 3 times as volatile as GDP, residential investments are more than twice as volatile as business investments and house prices are almost twice as volatile as GDP. In line with Dorofeenko, Lee, and Salyer (2014), we find that time-varying uncertainty is important for the latter result. The model further reproduces the empirically observed correlation between GDP and residential investments and between GDP and house prices. The correlations between residential investments and house

[1] Note that in a benchmark one sector model co-moving business and residential investments are a puzzle because the household intends to increase productive capital first. See also Kydland, Rupert, and Sustek (2016).

prices and between residential investments and business investments match the data in sign but are—at odds to the data—close to one.

An earlier analysis of risk premia in a production economy with housing, habits, and adjustments costs, which is similar to our extension of the Jermann (1998) model is presented in Jaccard (2011). However, different from our work and in contrast to JKKST and Flavin and Yamashita (2002), Jaccard (2011) considers data where the return on housing is markedly smaller than the return on equity. His empirical targets are based on Piazzesi, Schneider, and Tuzel (2007) who assume that the house price index grows with the price index of residential investments, whereas Davis and Heathcote (2007) and Knoll, Schularick, and Steger (2017) show that the main driver for increasing house prices are land prices. Moreover, different from the present paper Jaccard (2011) does not focus on returns on total risk. Lastly, Jaccard (2011) models superficial habits which have no intratemporal effect and the habit parameter is close to one.[2] The economic plausibility of both assumptions is questionable.

To the best of our knowledge, more general risk premia have not been investigated in production economies with disaster risk up to this date.

Favilukis, Ludvigson, and Nieuwerburgh (2017) study a two-sector production economy with aggregated and idiosyncratic income risk and use this framework in order to explain the boom-bust cycle in the first decade of this century. In their model, incomplete markets produce sizable risk premia for returns on equity and housing. While the model can match the Sharpe ratio of equity, the mean and the standard deviation of the return on equity turn out too small. Moreover, the return on housing is twice as large as the return on equity, which contrasts the data. Due to heterogeneity, there is no comparable measure for the volatility of returns on housing.

The risk usually associated with housing wealth is potentially of a more idiosyncratic nature than the risk from equity.[3] In the present paper, we do not consider such differences in the typical nature of risks. Similarly, the models abstract from other asset specific characteristics such as liquidity, transaction costs and search and matching frictions. Instead, we choose to face the aggregated data from JKKST throughout with a representative agent framework with complete markets. In this regard we understand our study as a first exploration of i) the asset pricing and business cycle characteristics which can already be explained within an elementary representative agent framework with complete markets, of ii) the characteristics for which such a framework becomes insufficient, and of iii) the reasons why a more sophisticated framework which helps to further differentiate between the assets is required for the characteristics in ii). Concerning i) we find that the model with disaster risk allows us to generate a Sharpe ratio which is substantially larger than the value previously confronted with for equity and which is close to the Sharpe ratio that is observed for housing. Moreover, the model can explain different means and volatilities of the risky assets while it still maintains a good fit to business cycle statistics. However, in regard to ii) the main shortcoming of the framework is that it cannot generate different Sharpe ratios of the risky assets. Different Sharpe ratios require different correlations between premia and the stochastic discount factor. Yet, in all of the models considered in the present paper, the return rates of the two risky assets are far too strongly correlated. We conclude, that further adjustments which help to disentangle this correlation are necessary.

From here on the papers reads as follows. In section 3.2 we first present the stylized facts on which we focus in the remainder of the paper. Section 3.3 presents and discusses the non-disaster

[2] Jaccard (2011) sets the habit parameter implicitly to one and only calibrates the habit persistence. With stationary variables the value of the habit parameter equals the reciprocal of the growth factor (=0.995).

[3] While JKKST report a standard deviation of 3.38 percent for the aggregated return on housing in the US data, Flavin and Yamashita (2002) as well as Landvoigt, Piazzesi, and Schneider (2015) find a standard deviation of the individual's return on housing of 14 percent. Hence, one potential approach to explain the different Sharpe ratios between equity and housing found in the aggregated data may be the idiosyncratic nature of the risk associated with housing.

economies, and section 3.4 introduces and discusses the economies with disaster risk. The paper concludes with section 3.5 and more detailed derivations are collected in the appendix.

3.2 Stylized facts

We start with the presentation of stylized facts which characterize historical data on business cycles, housing and asset prices and which the literature has identified as key facts that are commonly valid for most countries over longer time periods (see e.g. JKKST for asset prices and Davis and Nieuwerburgh (2015) for housing and business cycles). In Tables 1 and 2 we provide a summary of these stylized facts for the US (1970-2015), the UK (1969-2015), France (1980-2015), and Japan (1963-2015) while Appendix A provides the results for additional countries. Asset price statistics were computed from annual data from the JKKST database while business cycle statistics are shown for quarterly data from the OECD.stats library.1.

First, the upper part of Table 3.1 displays the mean return rates on bills, on equity, on housing and on total risk, and the standard deviations of the return rates are found in the lower part of the table. We observe a low risk-free return rate between 0.98 percent in Japan and 2.24 percent in France together with a low standard deviation (2.3-3.7). Note, however, that bills are not totally risk-free and, hence, only provide an upper bound proxy for the *true* risk-free return rate. The return on equity is between 5.86 percent in Japan and 9.61 percent in France and leads to equity premia between 4.88 percent and 7.37 percent. In all countries, the average return on housing turns out moderately smaller than the average return on equity, and housing premia between 3.54 percent in France and 5.44 percent in the UK can be observed. The difference between the two risky returns/premia is the smallest in Japan with just 0.32 percentage-points and the largest in France with 3.83 percentage-points. For the US and the UK the differences are 1.43 percentage-points and 1.00 percentage-points, respectively.[4] Moreover, in the US and the UK the return on total risk is approximately the average of the two risky return rates. In France the return on total risk is close to the smaller return on housing while in Japan the return on total risk exceeds the decomposed return rates on both risky assets.

While equity shows moderately larger returns than housing, on the downside the return rates on equity are two to four times as volatile as the return rates on housing. Both risky returns are least volatile in the US with standard deviations of 16.7 and 3.78, respectively, while the largest standard deviations are observed in France (24.11) for equity and in the UK (9.65) for housing. In all countries, the standard deviation of returns on total risk is also significantly lower than the standard deviation of returns on equity, and premia are almost identically as volatile as return rates. Finally, in all countries the Sharpe ratio of housing exceeds the Sharpe ratio of equity significantly, and the Sharpe ratio of total risk is close to the Sharpe ratio of housing. Summing up, we observe the following characteristics for return rates: i) a risk-free rate in the range of 1-2.2 percent together with a low volatility, ii) return rates on equity moderately larger than returns on housing, iii) premia on risky returns over 3 percent, iii) return rates and premia on equity which are at least twice as volatile as return rates and premia on housing and on total risk, and iv) a Sharpe ratio of housing significantly larger than the Sharpe ratio of equity and similar to the Sharpe ratio of total risk.

Second, Table 3.2 shows the stylized facts from the housing and the business cycle literature. We observe that GDP has a standard deviation of approximately 1.5-1.6 percent in the US, the UK and Japan while its standard deviation is slightly below 1 percent in France. In the US and the UK residential investments are twice as volatile as business investments while the difference

[4]The difference between the return rates in France is closer to the value in the other countries in the time periods chosen by JST (1963-2015 and 1870-2015). Our French data set starts in 1980 due to missing data for the business cycle statistics.

Table 3.1: Returns, premiums and second moments

	R_E	R_H	R_T	R_f	EP	HP	TP	SR_E	SR_H	SR_T
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27	0.36	1.01	0.75
UK	8.00	7.00	7.47	1.56	6.44	5.44	5.91	0.27	0.61	0.69
FRA	9.61	5.78	6.61	2.24	7.37	3.54	4.37	0.31	0.57	0.59
JPA	5.86	5.54	6.19	0.98	4.88	4.56	5.21	0.24	0.7	0.65
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_f)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$			
USA	16.71	3.78	6.90	2.31	16.47	4.41	7.00			
UK	23.41	9.64	8.44	3.73	24.27	8.88	8.62			
FRA	24.11	5.52	6.95	2.55	23.98	6.18	7.39			
JPA	20.15	6.53	8.10	2.53	19.94	6.47	8.03			

Notes: Mean percentage returns on equity (R_E), housing (R_H), total risk (R_T) and bills (R_f) as well as the equity premium (EP), the housing premium (HP), and the total risk premium (TP). The corresponding standard deviations $\sigma(X)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). Periods: USA 1970-2015, United Kingdom 1969-2015, France 1980-2015, and Japan 1963-2015. Data from JKKST, own calculations.

Table 3.2: Empirical business cycle statistics

	σ_{GDP}	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$r_{RESI}^{P_h}$	r_{RESI}^{BUSI}	r_{RESI}^{GDP}	$r_{GDP}^{P_h}$
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.64
UK	1.58	2.68	5.56	4.85	0.51	0.16	0.69	0.71
FRA	0.95	2.75	3.17	3.19	0.65	0.64	0.81	0.48
JPA	1.59	2.41	3.84	2.70	0.31	0.27	0.45	0.55

Notes: Business cycle statistics are from quarterly logged per capita hp-filtered (1600) data. σ_x is the standard deviation of x, r_y^x the correlation between x and y. RESI=residential investment, BUSI=non-residential investment, P_h house prices. Periods: USA: 1970-2015, , United Kingdom 1969-2015, France 1980-2015 Japan 1963-2015. Data: See Appendix 3.A, own calculations.

between the two volatilities is moderately smaller in Japan and significantly smaller in France.[5] In all four countries the standard deviation of business investment lies between 2.4 and 2.9 percent and house prices are pro-cyclical. GDP, house prices, residential and business investment co-move and the lowest correlation is observed between business and residential investments. In short, sub-aggregates and house prices co-move pro-cyclically. Usually the literature additionally considers lagged cross-correlations with residential investments since residential investments lead the business cycle in the US. However, Kydland, Rupert, and Sustek (2016) show that this fact is unique to the US and Canada which is why we omit lead-lag-patterns here.

Next to the four countries discussed in this section, Appendix 3.A shows that we also observe the same stylized facts in most other countries.

3.3 Economies with non-disaster risk

In this section, we add housing to influential approaches to explain the equity premium puzzle in production economies. We start with an adaption of the Jermann (1998) model with habit formation and capital adjustment costs in line with the 'q' theory (model A). We then extend the model by housing (model B). In a next step, we separate the production of residential investments from the production of the consumption good and business investments. The two sectors are subject to limited sectoral capital mobility similar to Boldrin, Christiano, and Fisher (2001) and Fehrle (2019). We consider the cases of exogenous labor (model C), endogenous and

[5]For most continental European countries we observe the same relation as in France.

fully mobile labor (model D), and endogenous labor subject to limited sectoral labor mobility (model E).

3.3.1 Housing with Jermann (1998)

Model A: Our study starts with the seminal work of Jermann (1998) with habits in utility, adjustment costs in capital and exogenous labor decisions. Our variation of the model deviates from its original treatment only in that we consider exogenous habits that are out of the household's control.

Model B: We proceed to extend the Jermann (1998) model by housing. The household draws utility from housing H_t and consumption C_t, and both are subject to habit formation. Habits X_{ht}, $X \in \{C, H\}$, are exogenous and evolve according to $X_{ht} = \chi_X X_{t-1}$. Labor supply remains exogenous. Output Y_t is produced with capital K_t and is subject to labor augmenting technical progress growing at the rate a_y in the long run, and to productivity shocks Z_t following an AR(1)-process, $\ln Z_{t+1} = \rho_y \ln Z_t + \epsilon_{t+1}$, $\epsilon_t \sim \text{iidN}(0, \sigma_y^2)$. Consumption, business investment I_t, and residential investment D_t are homogeneous goods. We stick to the assumption of capital adjustment costs as in Jermann (1998).[6] A fixed factor normalized to one, namely land, affects the transformation from residential investment to new houses. The planner's problem in a centralized economy therefore reads as follows:[7]

$$
\begin{aligned}
\max_{C_t, I_t, D_t, K_{t+1}, H_{t+1}} \quad & U_0 = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{((C_t - C_{ht})^{\mu_c}(H_t - H_{ht})^{\mu_h})^{1-\eta} - 1}{1-\eta}, \\
\text{s.t.} \quad & Y_t = a_y^{(1-\alpha_y)t} Z_t K_t^{\alpha_y}, \\
& Y_t = C_t + I_t + D_t, \\
& K_{t+1} = (1-\delta_k)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t, \\
& H_{t+1} = (1-\delta_h)H_t + D_t^{1-\phi},
\end{aligned}
\tag{3.3.1}
$$

where $\eta, \alpha_y, a_y, \mu_c, \mu_h > 0$, $\mu_c + \mu_h = 1$, $0 < \beta, \delta_k, \delta_h < 1$, and $\Phi(x) := \frac{\varphi_1}{1-\kappa} x^{1-\kappa} + \varphi_2$, $\varphi_1 > 0$, $\varphi_2 \in \mathbb{R}$.

We follow Davis and Heathcote (2005) and define GDP by $\text{GDP}_t = Y_t + MRS_t^{H,C} H_t$, where $MRS_t^{H,C} = (\mu_h/\mu_c)(C_t - C_{ht})/(H_t - H_{ht})$ denotes the marginal rate of substitution between housing and consumption so that its product with the current housing stock yields the implicit rent from housing. Finally, the return rate $R_{E,t+1}$ on investment in productive capital, the return rate $R_{H,t+1}$ on housing, the return rate $R_{T,t+1}$ on total risk, and the risk-free rate $R_{f,t}$ are given by

$$
\begin{aligned}
1 + R_{E,t+1} &= \frac{r_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta_k + \Phi(\frac{I_{t+1}}{K_{t+1}}))}{q_t} = \frac{\alpha Y_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}}, \\
1 + R_{H,t+1} &= \frac{MRS_{t+1}^{H,C} + (1-\delta_h)P_{ht+1}}{P_{ht}}, \\
1 + R_{T,t+1} &= \frac{\alpha Y_{t+1} - I_{t+1} + q_{t+1}K_{t+2} + (MRS_{t+1}^{H,C} + (1-\delta_h)P_{ht+1})H_{t+1}}{q_t K_{t+1} + P_{ht+1}H_{t+1}},
\end{aligned}
$$

[6] In an earlier version of the paper we assumed investment adjustment costs as in Christiano, Eichenbaum, and Evans (2005). Fehrle (2019) shows for the Davis and Heathcote (2005) framework that these adjustment costs account better for the lag pattern of business investment. However, to remain in line with Jermann (1998), Gourio (2012) and our disaster risk framework, we changed to capital adjustment costs. Besides the lead-lag structure, which is beyond the scope, changes are minor.

[7] See Appendix 3.C for details.

$$1+R_{f,t} = \frac{\Lambda_t}{\beta \mathbb{E}_t \Lambda_{t+1}},$$

where $q_t = 1/\Phi'(I_t/K_t)$ is Tobin's q, $r_t = \alpha_y(Y_t/K_t)$ is the rental rate of capital which equals the marginal product of capital, $\Lambda_t = \mu_c(C_t - C_{ht})^{\mu_c(1-\eta)-1}(H_t - H_{ht})^{\mu_h(1-\eta)}$ is the marginal utility of consumption and $P_{ht} = D_t^{\phi}/(1-\phi)$ denotes house prices which equal the reciprocal of the residential investment's marginal rate of production of new houses.

Calibration A: We identify one period in the model with one quarter in the data and closely follow the calibration in Jermann (1998). More precisely, we set the coefficient of relative risk aversion to $\eta = 5$, the elasticity of capital in the production function to $\alpha_y = 0.36$ and the quarterly trend growth rate to $a_y = 1.005$ as in Jermann (1998). We slightly deviate from the value of $\delta_k = 0.025$ used in Jermann (1998) and, in foresight of model B, instead adjust the depreciation rate of capital from Nguyen (2018), who strips down the Davis and Heathcote (2005) model, to quarterly data which yields $\delta_k = 0.022$. The autocorrelation parameter and the conditional standard deviation of the AR(1)-process governing productivity are pinned down to ρ_y=0.95 and σ_y=0.01. In line with Jermann (1998), we choose the remaining parameters of the model, i.e. the household's time preference β, the habit parameter χ_c, and the parameter κ controlling the elasticity of the investment capital ratio with respect to Tobin's q, in such way to closely replicate the risk-free rate, the equity premium and the relative volatility of business investment to GDP from the US data. We minimize the (unweighted) sum of squared deviations between the targets in the model and the values in the data over a grid covering $\beta^{\star} := \beta a_y^{1-\eta} \in [0.99; 0.999]$, $\chi_c \in [0; 0.95]$ and $\kappa \in [0; 6.25]$ where the number of grid-point is 10, 10 and 50, respectively. The resulting values are summarized in Column A of Table 3.3.

Calibration B: In order to keep the different variations of the model comparable and in order to emphasize the effects of introducing housing into the Jermann (1998) model, all parameters from model A also remain at the same values in model B.[8] In particular, we do not re-optimize the previously "free" parameters for model B in order to match the (additionally available) targets. However, re-optimizing would not change the following main results.

We calibrate the additional parameters from housing in model B as follows. First, we also borrow the depreciation rate of housing $\delta_h = 0.009$ from the same source as we did δ_k. Second, we follow Grossmann, Larin, Löfflad, and Steger (2019) and pin down the weights μ_c and μ_h of consumption and of housing in the consumption bundle such way that the ratio of expenditures on housing to total consumption is 19 percent on the balanced growth path and so that $\mu_c + \mu_h = 1$ holds. Third, the habit parameter for housing is set to the same high value of $\chi_h = 0.95$ as for consumption. Finally, we take the value of the land parameter $\phi = 0.106$ from Davis and Heathcote (2005).

Results: The return rates as well as the business cycle statistics for our variation of the Jermann (1998) model (row A) and for the model extended by housing (row B) are summarized in tables 3.4 and 3.5. We compute the annualized mean return rates and the annualized standard deviation of return rates from a simulation of 100,000 periods. The second moments of the business cycle are reported as the average outcome from 100 repeated simulations of HP-filtered time series of the model's equilibrium outcomes, each for 180 periods. The model solution is obtained from a second-order perturbation method.

First, as shown by Jermann (1998), model A is able to generate a sizeable equity premium and a risk-free rate which are close to the values observed in the data. Moreover, we can also closely replicate the volatility of business investments relative to the volatility of GDP. On the other hand, the return rates, especially the risk-free rate, turn out too volatile in the model.

However, once housing is introduced into the model, all risk premia—on equity, housing

[8] Note however, that η now is the coefficient of relative risk aversion with respect to the composite good and no longer with respect to consumption only. Moreover, now $\beta^{\star} := \beta a^{(\mu_c + (1-\phi)\mu_h)(1-\eta)}$.

Table 3.3: Calibration

	A	B	C	D / E	Description
$\beta^{\star}$	0.994	0.994	0.999	0.999	discount factor
η	5	5	5	5	coefficient of relative risk aversion
$\mu_c^{*)}$	–	0.81	0.81	0.53	weight of consumption in composite good
$\mu_h^{*)}$	–	0.19	0.19	0.12	weight of housing in composite good
χ_c	0.95	0.95	0.825	0.825	habit parameter of consumption
χ_h	–	0.95	0.825	0.825	habit parameter of housing
χ_n	–	–	–	0.95	habit parameter of leisure
a_y	1.005	1.005	1.005	1.005	growth rate (y sector)
a_d	–	–	1.002	1.002	growth rate (d sector)
ϕ	–	0.106	0.106	0.106	share of land in housing
α_y	0.36	0.36	0.25	0.25	capital share in production (y sector)
α_d	–	–	0.20	0.20	capital share in production (d sector)
κ_y	4.05	4.05	6.25	6.25	elasticity of Tobin's q (y sector)
κ_d	–	–	1.25	1.25	elasticity of Tobin's q (d sector)
δ_k	0.022	0.022	0.022	0.022	rate of capital depreciation (y sector)
δ_h	–	0.009	0.009	0.009	rate of housing depreciation (d sector)
σ_y	0.01	0.01	0.0094	0.0094	conditional standard deviation of log TFP (y sector)
ρ_y	0.95	0.95	0.966	0.966	autocorrelation of log TFP (y sector)
σ_d	–	–	0.0172	0.0172	conditional standard deviation of log TFP (d sector)
ρ_d	–	–	0.923	0.923	autocorrelation of log TFP (d sector)

Notes: $^{*)}$ Endogenous by the model. **A**: Jermann (1998) adaption. **B**. A + Housing in utility. **C**. B + two sectors. **D**: C + endogenous labor. **E**: D + limited sectoral labor mobility.

and total risk—turn out close to zero, and the volatility of return rates is reduced drastically. Introducing housing into the model provides the household with an option to better insure against fluctuations in his marginal utility in the same way as discussed by Uhlig (2007) for endogenous labor decisions. Since consumption and residential investment are homogeneous, the household is now able to reduce residential investments in favor of consumption in response to negative productivity shocks. The relatively small elasticity ($\mu_h = 0.14$) combined with a small depreciation rate of housing ($\delta_h = 0.009$) favor the household's possibilities to smooth his consumption bundle across states with different realizations of the shock. In consequence, the stochastic discount factor becomes far less volatile so that risk premia almost disappear. Moreover, the household's efforts to smooth his consumption bundle by adequately adjusting residential investment and consumption also show up in the second moments of the business cycle. The volatility of residential investment in the model is twice as large as in the data and the demand of residential goods moves procyclical. Further, residential investments are positively correlated with house prices and the other variables considered.

3.3.2 Moving to Boldrin, Christiano, and Fisher (2001)

Sectoral frictions with exogenous labor supply: In the previous subsection, the household can nearly perfectly hedge against consumption fluctuations since the marginal rate of transformation between residential investment and consumption was one. We restrict this option in the following by moving to a two-sector model—separating production of the residential good from production of the consumption good—with frictions in factor mobility. In particular, capital is assumed immobile between the two sectors. The resulting model is similar to Nguyen (2018). We start with exogenous labor supply before discussing the effects of endogenous labor supply and labor supply which is contracted sector-specifically one period ahead as proposed by Boldrin, Christiano, and Fisher (2001).

Table 3.4: Returns, premiums and second moments

	R_E	R_H	R_T	R_f	EP	HP	TP	SR_E	SR_H	SR_T
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27	0.36	1.01	0.75
					Model					
A	7.66	–	–	1.55	6.05	–	–	0.25	–	–
B	4.61	4.39	4.52	4.26	0.34	0.13	0.26	0.07	0.07	0.07
C	4.71	4.45	4.56	0.38	4.31	4.06	4.16	0.21	0.21	0.21
D	2.57	2.47	2.51	2.05	0.51	0.42	0.46	0.07	0.07	0.07
E	4.88	4.61	4.72	1.50	3.34	3.07	3.18	0.18	0.18	0.18
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_f)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$			
USA	16.71	3.78	6.9	2.31	16.47	4.41	7.00			
					Model					
A	25.17	–	–	6.19	24.22	–	–			
B	5.01	2.07	3.79	0.73	4.95	1.93	3.71			
C	21.83	20.73	21.19	8.08	20.26	19.06	19.57			
D	7.18	5.95	6.44	1.47	7.03	5.76	6.27			
E	22.60	21.45	21.92	13.39	18.19	16.72	17.33			

Notes: Mean percentage returns on equity (R_E), housing (R_H), total risk (R_T) and bills (R_f), as well as the equity premium (EP), the housing premium (HP), and the total risk premium (TP). The corresponding standard deviations $\sigma(X)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). We employ a second order perturbation and simulated time series with 100,000 periods. **A**: Jermann (1998) adaption. **B**: A + Housing. **C**: B + two sectors. **D**: C + endogenous labor. **E**: D + limited sectoral labor mobility.

Model C: The household's utility and the law of motion of the housing stock remain the same as in (3.3.1). The model economy consists of two sectors indexed by y and d. The sector y produces a homogeneous consumption and business investment good while the residential investment good is produced in sector d. Sector-specific technical progress grows at the rate a_x in sector x, $x \in \{y, d\}$, and both sectors are subject to sector-specific and uncorrelated productivity shocks Z_{xt} governed by AR(1)-processes, $\ln Z_{x,t+1} = \rho_x \ln Z_{x,t} + \epsilon_{x,t+1}$, $\epsilon_{x,t} \sim \text{iidN}(0, \sigma_x^2)$. The household is confronted with capital adjustment costs in both sectors and, once installed, capital

Table 3.5: Simulated business cycle statistics I

	σ_{GDP}	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$r_{RESI}^{P_h}$	r_{RESI}^{BUSI}	r_{RESI}^{GDP}	$r_{GDP}^{P_h}$
					Data			
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.64
					Model			
A	1.25	2.91	–	–	–	–	–	–
B	0.96	0.74	11.63	1.16	0.98	0.98	0.98	0.96
C	2.27	1.04	0.96	4.42	−0.03	0.00	0.08	0.97
D	1.45	0.69	4.25	2.37	0.65	0.77	0.85	0.90
E	1.96	0.96	2.10	3.97	−0.06	0.08	0.36	0.89

Notes: σ_x is the standard deviation of x, r_y^x the correlation between x and y. Business cycle statistics from HP-filtered (1600) times series. We employ a second order perturbation and report the average outcomes from repeated simulations with 180 periods. **A**: Jermann (1998) adaption. **B**: A + Housing. **C**: B + two sectors. **D**: C + endogenous labor. **E**: D + limited sectoral labor mobility.

is totally immobile. The household's problem in a centralized economy reads as follows:

$$\begin{aligned}
\max_{C_t,D_t,I_{yt},I_{dt},K_{yt+1},K_{dt+1},H_{t+1}} \quad & U_0 = \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{((C_t - C_{ht})^{\mu_c}(H_t - H_{ht})^{\mu_h})^{1-\eta} - 1}{1-\eta}, \\
\text{s.t.} \quad & Y_t = a_y{}^{(1-\alpha_y)t} Z_{yt} K_{yt}^{\alpha_y}, \\
& Y_t = C_t + I_t, \\
& D_t = a_d{}^{(1-\alpha_d)t} Z_{dt} K_{dt}^{\alpha_d}, \\
& I_t = I_{dt} + I_{yt}, \\
& K_{yt+1} = (1-\delta_k)K_{yt} + \Phi_y\left(\frac{I_{yt}}{K_{yt}}\right)K_{yt}, \\
& K_{dt+1} = (1-\delta_k)K_{dt} + \Phi_d\left(\frac{I_{dt}}{K_{dt}}\right)K_{dt}, \\
& H_{t+1} = (1-\delta_h)H_t + D_t^{1-\phi}.
\end{aligned} \tag{3.3.2}$$

where $\eta, \alpha_y, \alpha_d, a_y, a_d, \mu_c, \mu_h > 0$, $\mu_c + \mu_h = 1$, $\beta, \delta_k, \delta_h \in (0,1)$, $\Phi_y(x) = \varphi_{y,2} + \frac{\varphi_{y,1}}{1-\kappa_y} x^{1-\kappa_y}$, and Φ_d analogously.

GDP is now defined by $\text{GDP}_t = Y_t + P_{dt}D_t + MRS_t^{H,C} H_t$ where P_{dt} is the relative price of residential investment goods. The return on housing remains the same as before but with $P_{ht} = P_{dt}D_t^{\phi}/(1-\phi)$, while the return on equity is the weighted sum of the return on capital in the two sectors, i.e. with the obvious adaption of notation from the one-sector model

$$1 + R_{E,t+1} = \frac{\alpha_y Y_{t+1} - I_{y,t+1} + q_{y,t+1}K_{y,t+2} + \alpha_d P_{d,t+1}D_{t+1} - I_{d,t+1} + q_{d,t+1}K_{d,t+2}}{q_{y,t}K_{y,t+1} + q_{d,t}K_{d,t+1}}.$$

The return on total risk is adjusted in an analogous way.

Calibration: The parameters η, ϕ, δ_k and δ_h remain at the same values they were previously set to in model B. Likewise, the weights μ_c and μ_h of consumption and housing in the household's utility are still pinned down by imposing that the ratio of expenditures on housing to total consumption is 19 percent on the balanced growth path. In order to take the two sector framework into account, we assume the same capital shares, α_y and α_d, as in Nguyen (2018). Moreover, we also take the autocorrelation parameters ρ_y and ρ_d of shocks to productivity in both sectors from Nguyen (2018). The standard deviations of innovations are chosen in such way that their ratio is kept the same as in Nguyen (2018) while the level is adjusted to reproduce a standard deviation of GDP comparable to models A and B and to the data. As already noted, we abstract from technology spillovers.[9] While Nguyen (2018) does not consider long-run growth, we choose $a_y = 1.005$ and $a_d = 1.002$ to match the annual output growth rates in the two sectors as reported by Davis and Heathcote (2005).

The remaining parameters are set again in such way that the (unweighted) sum of squared deviations between our targets in the model and in the data is minimized. The list of targets now includes the risk-free rate, the equity premium, the housing premium as well as the relative standard deviations and the correlations from the business cycle statistics in Table 3.2, all for US data. Our grid covers $\beta^\star := \beta a_y^{(\mu_c+(1-\phi)\mu_h\alpha_d)(1-\eta)} a_d^{(1-\phi)(1-\alpha_d)\mu_h(1-\eta)} \in [0.99, 0.999]$, $\chi_c, \chi_h \in [0.7, 0.95]$, and $\kappa_y, \kappa_d \in [0.625, 6.25]$ and is built-up from $10\times5\times5\times10\times10$ grid-points. A summary of the model's calibration is given in column C of Table 3.3.

Results: The return rates and business cycle statistics in the two-sector model are shown in row C of tables 3.4 and 3.5. First, restricting the household's option to smooth his consumption

[9] Spillovers in Nguyen (2018) are negligible. Fehrle (2019) makes the case for preventing spill-overs and correlated shocks.

bundle by switching from residential investments to consumption has the desired effect on asset prices. Compared to model B, risk premia in the model again increase substantially. The model generates an equity premium of 4.31 percent which is about one and a half percentage-points below the value found in the data while the premium on housing in the model is moderately lower at 4.06 percent and is a half percentage-point below its empirical counterpart. Similar to the empirical findings, the model yields a premium of total risk in between the two premia of equity and housing. The model also reproduces a low risk-free rate but fails to explain the observed volatilities of asset prices. The standard deviation of the risk-free rate exceeds its empirical value by a factor of four, return rates on housing turn out too volatile by a factor of almost six, and the volatility of returns on total risk is too large by a factor of almost three. The model, hence, cannot explain a Sharpe ratio of housing which is markedly larger than that of equity.

The restriction of the household's option in the allocation between consumption and residential investments has a negative effect on the business cycle statistics. While in model B the household's preference to smooth the consumption bundle induces procyclical co-movement in the demand of residential goods, the positive correlation between house prices and residential investment now disappears. Moreover, the assumption of uncorrelated shocks in the two sectors prevents co-movements between residential and business investment. Since consumption and business investments account for the largest part of GDP, residential investments and GDP fluctuate almost uncorrelatedly.

Endogenous labor supply: Allowing the household to adjust labor supply in response to productivity shocks, again opens a channel which admits to smooth the consumption bundle more evenly across different states of shocks. As pointed out by Uhlig (2007), risk premia in the model should suffer.

Model D: Hours worked in the two sectors, N_{yt} and N_{dt}, augment the production functions and aggregated hours $N_t = N_{dt} + N_{yt}$ cannot exceed the time endowment of the household which is normalized to one. Accordingly, leisure $(1-N_t)$ is added to the household's utility function which is parameterized as in Davis and Heathcote (2005) but extended by habit formation in leisure equivalent to consumption and housing, i.e. $N_{ht} := 1-\chi_n(1-N_{t-1})$. The changes to the household's problem from (3.3.2) are as follows

$$\begin{aligned}
\max_{\dots,N_{yt},N_{dt}} \; & U_0 = \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{((C_t-C_{ht})^{\mu_c}(H_t-H_{ht})^{\mu_h}((1-N_t)-(1-N_{ht}))^{\mu_n})^{1-\eta}-1}{1-\eta}, \\
\text{s.t.} \quad & Y_t = Z_{yt}K_{yt}^{\alpha_y}(a_y^t N_{yt})^{1-\alpha_y}, \\
& Y_t = C_t + I_t, \\
& D_t = Z_{dt}K_{dt}^{\alpha_d}(a_d^t N_{dt})^{1-\alpha_d}, \\
& N_t = N_{dt} + N_{yt}, \\
& N_t \leq 1 \\
& \dots
\end{aligned} \tag{3.3.3}$$

where now $\mu_c, \mu_h, \mu_n > 0,\ \mu_c + \mu_h + \mu_n = 1$.

Calibration: Again, in order to place emphasis on the effects of introducing endogenous labor decisions to the model, all parameters from model C remain at the same values as before. We only adjust the weights μ_c, μ_h and μ_n in the household's utility in such way that i) the housing expenditures remain at 19 percent of total consumption expenditures and ii) he works one third of his time endowment on average. Moreover, the habit parameter χ_n is set to its upper bound 0.95 of plausible values. Column D/E of Table 3.3 outlines the calibration.

Results: Row D of Table 3.4 confirms the already expected consequences of endogenous labor supply for the return rates in the model. Compared to model C, the return rates on equity, housing, and total risk, decrease and become significantly less volatile. In consequence, risk premia drastically fall by a magnitude of order.

Endogenous labor supply reintroduces the possibility for adjustments in the allocation of the consumption bundle after the shock's realization. The household is able to adjust his working hours intersectorally and can shift conveniently between consumption and residential investments. In consequence, the discussed demand effect for residential investment recurs as can be seen in row D of Table 3.5. The model can explain the volatilities of residential investment and house prices fairly well while business investment remains too involatile. Moreover, the model can also generate the positive correlations between house prices, residential investment, and GDP found in the data. Yet, residential and business investment are correlated too strongly.

Limited labor mobility: Two well-known extensions that help to revive risk premia when labor decisions are endogenous are limited sectoral mobility as described by Boldrin, Christiano, and Fisher (2001) and wage rigidities as proposed by Uhlig (2007). To keep in line with the present framework of limited factor mobility, we focus on the former.

Model E: The household is now unable to adapt his labor supply in response to technology shocks but is committed to working hours that are contracted sector-specifically one period ahead. Nothing else changes so that the household's problem remains as in (3.3.3) with the exception that he now optimizes with regard to N_{yt+1} and N_{dt+1} while taking N_{yt} and N_{dt} as given in any period t.

Calibration: We stick to the calibration in column D/E of Table 3.3 from the previous model D with a frictionless labor market.

Results: Return rates from the two-sector model with limited labor mobility are summarized in row E of Table 3.4. Limited labor mobility provides a mixture of the two previous cases with exogenous labor supply in model C and with endogenous and frictionless labor supply in model D. Hence, risk premia increase significantly compared to model D but remain below the values from model C. Moreover, the standard deviation of the risk-free rate turns out too large by a factor of almost six and the return rates on housing and on total risk are more than three times too volatile.

Table 3.5 shows in its row E that the model can generate positive correlations between business and residential investment, between GDP and residential investment and between GDP and house prices which are all close to the values in the data. However, the attempt to explain risk premia by shutting down the channel that enables the household to smooth his consumption bundle comes at the cost of too involatile residential investment which is no longer positively correlated with house prices.

3.3.3 Summary and discussion

In the classic Jermann (1998) model, habits increase the household's desire to smooth consumption of the composite good. However, if the model is extended by housing in a one sector framework, optimal adjustment of the allocation of output to consumption and residential investment enables the household to insure himself more conveniently against fluctuations in the consumption bundle. A small elasticity of housing in the consumption bundle and the rather small depreciation rate of housing favor the behavior. A similar argument holds in a multi-sector framework with perfect labor markets where the household can adapt the allocation of hours worked in each sector in response to productivity shocks. While this option implies that the marginal utility does not fluctuate enough between different realizations of the shock and therefore yields risk premia close to zero, it induces, on the other hand, a demand effect

Table 3.6: Risk premia components

	$\frac{2\sqrt{\mathrm{Var}[M_{t+1}]}}{(\mathbb{E}[M_{t+1}])^4}$	SR_E	SR_H	$r_{M,\mathrm{EP}}$	$r_{M,\mathrm{HP}}$	$r_{\mathrm{EP,HP}}$
			Data			
USA	–	0.36	1.01	–	–	0.19
			Model			
B	0.07	0.07	0.07	−0.99	−0.99	1.00
C	0.23	0.21	0.21	−0.92	−0.92	1.00
D	0.08	0.07	0.07	−0.98	−0.96	0.99
E	0.23	0.18	0.18	−0.80	−0.80	1.00

Notes: SR_E : annualized Sharpe ratio of equity, SR_H : annualized Sharpe ratio of housing, $r_{X,Y}$: correlation between variables X and Y where M_{t+1} : stochastic discount factor, $EP_{t+1} := R_{E,t+1} - R_{f,t}$: ex-post equity premium, $HP_{t+1} := R_{H,t+1} - R_{f,t}$: ex-post housing premium.

which results in positive correlations between residential investment and house prices and in standard deviations in business cycle statistics that are close to the data.

Risk premia can be increased through sectoral frictions as e.g. limited capital and labor mobility. Yet, this comes at the cost of losing the empirical co-movements of residential investment. Therefore, we conclude that the present framework cannot simultaneously reproduce asset pricing statistics and business cycle statistics as observed in the data. Moreover, the models fail to explain the different volatilities and Sharpe ratios between the two risky assets throughout. In all models and contrary to the data, the mean as well as the standard deviation of returns on total risk are the weighted averages from the returns on equity and housing.

In order to provide some additional reasoning for the models' failures in regard to asset price statistics, note that the models' Euler equations together with the definition of the risk-free rate imply

$$\mathbb{E}_t\left[M_{t+1}(R_{E,t+1} - R_{f,t})\right] = \mathbb{E}_t\left[M_{t+1}(R_{H,t+1} - R_{f,t})\right] = 0,$$

where M_{t+1} denotes the models' respective stochastic discount factor. Taking unconditional expectations, the equality also holds unconditionally for the models' stationary distributions. Hence, for both assets $X \in \{E, H\}$,

$$\begin{aligned}\mathbb{E}[M_{t+1}]\,\mathbb{E}\left[R_{X,t+1} - R_{f,t}\right] &= -\mathrm{Cov}\left[M_{t+1}, R_{X,t+1} - R_{f,t}\right] = \\ &= -\mathrm{Corr}\left[M_{t+1}, R_{X,t+1} - R_{f,t}\right]\sqrt{\mathrm{Var}\left[R_{X,t+1} - R_{f,t}\right]}\sqrt{\mathrm{Var}[M_{t+1}]},\end{aligned}$$

or equivalently for the Sharpe ratio

$$\mathrm{SR}_X := \frac{\mathbb{E}\left[R_{X,t+1} - R_{f,t}\right]}{\sqrt{\mathrm{Var}\left[R_{X,t+1} - R_{f,t}\right]}} = -\frac{\sqrt{\mathrm{Var}[M_{t+1}]}}{\mathbb{E}[M_{t+1}]}\mathrm{Corr}\left[M_{t+1}, R_{X,t+1} - R_{f,t}\right]. \tag{3.3.4}$$

The first factor on the right hand side defines an upper bound and is common to both assets, while different correlations between risk premia and the stochastic discount factor between the two assets are necessary in order to explain different Sharpe ratios. More precisely, in order to match the different Sharpe ratios observed in the data, the correlation between premia on housing and the stochastic discount factor must be (in absolute value) approximately 3 times as large as the correlation between premia on equity and the stochastic discount.

We summarize the decomposition of the Sharpe ratios provided by equation (3.3.4) in Table 3.6. First, we observe that in all models the two risky return rates are almost perfectly correlated and, hence, the correlations with the stochastic discount factor are nearly identical. By (3.3.4), the Sharpe ratios of the two risky assets must also coincide. Although models C and E can replicate the empirically observed Sharpe ratio of equity fairly well, we can conclude that they achieve this result in an unfitting way. In order to leave room for a significantly larger Sharpe ratio of housing, the correlation between the stochastic discount factor and the premia on equity would have to be substantially smaller (in absolute value). The smaller correlation $r_{M,\mathrm{EP}}$ would then have to be offset with a larger standard deviation of the stochastic discount factor to keep the equity premium and its volatility the same, and less volatile premia on housing would be necessary in order to still match their mean. The models B and D, which fail to generate sizeable risk premia and Sharpe ratios, suffer from a too low volatility of the stochastic discount factor—the agent can adjust his decisions sufficiently well in response to shocks in order to keep fluctuations in his marginal utility small.

Moreover, the nearly perfect correlation between the return rates of the two risky assets also implies that the mean *and* the standard deviation of the total portfolio are the weighted averages of the two assets. Contrary to the observations from the stylized facts, the Sharpe ratio of total risk must coincide with the Sharpe ratios of the two risky assets.[10]

Fehrle (2019) discusses the implications of a larger share of land in the production of new houses. He shows that in the in the Davis and Heathcote (2005) framework the ability to account for co-moving economic activity, especially for the correlation between residential investment and house prices, can be improved. We follow Fehrle (2019) and repeat our computations for $\phi = 0.3$ which is the upper bound considered by Fehrle (2019). The results are summarized in Appendix 3.B.[11] We find that improvements in the business cycle statistics are only marginal and effects on asset return statistics are ambiguous.

3.4 Housing with disaster risk

In this section we move to another popular approach to explain risk premia. We combine an otherwise standard RBC model with housing and with key elements from the literature on economic disasters. The model is based on Gourio (2012). It is extended by housing and features a time-varying risk for disasters which reduce productivity and which also partly destroy the stocks of productive capital and residential structures. We choose to keep the model as simple as possible and provide easily traceable insights of the model's mechanisms instead of a richer framework that would supply more degrees of freedom to match the data. A more detailed presentation of the model, including our solution method, is delegated to Appendix 3.D.

3.4.1 Model

The basic framework of the model follows the one-sector model from the previous section. The household derives utility from a composite good $\tilde{C}_t$ that is represented by a Cobb-Douglas

[10] For any non-stochastic share $w \in (0,1)$ of equity in total wealth, if $r_{EP,HP} = 1$, then

$$\sigma(TP) = (w^2\sigma^2(EP) + (1-w)^2\sigma^2(HP) + 2w(1-w)r_{EP,HP}\sigma(EP)\sigma(HP))^{\frac{1}{2}} = w\sigma(EP) + (1-w)\sigma(HP),$$

and, if additionally $SR_E = SR_H$, then also

$$SR_T = \frac{w\sigma(EP)}{w\sigma(EP) + (1-w)\sigma(HP)} SR_E + \frac{(1-w)\sigma(HP)}{w\sigma(EP) + (1-w)\sigma(HP)} SR_H = SR_E = SR_H.$$

[11] Favilukis, Ludvigson, and Nieuwerburgh (2017) proceed similarly by setting the land share equal 0.25.

aggregate consisting of consumption C_t, housing H_t and leisure $1-N_t$, i.e.

$$\tilde{C}_t := C_t^{\mu_c} H_t^{\mu_h} (1-N_t)^{1-\mu_c-\mu_h}.$$

We assume that the household's preferences over streams of the composite good are described by a recursive utility function, as introduced by Epstein and Zin (1989) and Weil (1989), of the form

$$\tilde{V}_t = \left[(1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta (\mathbb{E}_t \tilde{V}_{t+1}^{1-\gamma})^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}},$$

where ψ is the household's elasticity of intertemporal substitution (EIS) and γ is the coefficient of relative risk aversion (RRA). Note however that γ and ψ describe the household's RRA and EIS with respect to the composite good $\tilde{C}$. Since the composite good aggregator is of the Cobb-Douglas type, the consumption-based RRA is given by $\mu_c \gamma$ and the consumption-based EIS reads $\frac{1}{1-\mu_c(1-1/\psi)}$.[12] For easier notation we define $V_t := \tilde{V}_t^{1-1/\psi}$ which satisfies the recursion

$$V_t = (1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta (\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}},$$

where we use, similar to Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012), the notation

$$\theta := 1 - \frac{1-\gamma}{1-\frac{1}{\psi}}.$$

In the case where $\theta = 0$, the RRA equals the reciprocal of the EIS and the household's utility reduces to the 'classical' expected discounted sum of within period CRRA utilities. Hence, θ can also be interpreted as the deviation from this 'classic' case.

Output Y_t is produced with the help of capital K_t and labor N_t according to the Cobb-Douglas production function $Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha}$ where A_t denotes labor augmenting technological progress which grows stochastically as will be outlined below. We stick to the assumption that investments in the productive capital stock are met with adjustment costs as in Jermann (1998). Output is allocated between the homogenous goods consumption, business investments, and investments in residential structures. Residential structures must be combined with land, which acts as adjustment costs to residential investments, before entering the stock of houses.

Additionally, the economy faces a great disaster risk. Disasters are introduced through an exogenous shock in form of a binary variable b_t which indicates disasters in case of $b_t = 1$ while $b_t = 0$ in normal times. Following Gourio (2012), disasters appear with time-varying probability and size. More specifically, we assume that

$$P(b_{t+1} = 1 | b_t = 0) = \min\{p_t, 1\}, \; P(b_{t+1} = 0 | b_t = 0) = 1 - \min\{p_t, 1\}$$

where the log of p_t follows an AR(1)-process

$$\ln p_{t+1} = (1-\rho_p)\ln \bar{p} + \rho_p \ln p_t + \epsilon_{p,t+1}, \; \epsilon_{p,t} \sim \text{iidN}(0, \sigma_p^2). \tag{3.4.1}$$

Additionally, disasters remain persistent with probability no less than $q \in (0,1)$ so that

$$P(b_{t+1} = 1 | b_t = 1) = \max\{q, \min\{p_t, 1\}\}, \; P(b_{t+1} = 0 | b_t = 1) = 1 - \max\{q, \min\{p_t, 1\}\}.$$

[12] See also Swanson (2012) and Heiberger and Ruf (2019).

On the one hand, disasters result in a decline of productivity by the factor $1-e^{\omega_{t+1}}$ so that technology grows stochastically according to

$$\begin{aligned} A_{t+1} &= A_t a e^{z_{t+1}+\omega_{t+1}b_{t+1}}, \\ z_{t+1} &= \rho_z z_t + \epsilon_{z,t+1},\ \epsilon_{z,t} \sim \text{iidN}(0,\sigma_z^2). \end{aligned}$$

On the other hand, disasters also result in the destruction of a fraction $1-e^{\omega_{t+1}}$ of the stocks of capital and residential structures, i.e.

$$\begin{aligned} H_{t+1} &= e^{\omega_{t+1}b_{t+1}(1-\phi)} \underbrace{\left((1-\delta_h)H_t + D_t^{1-\phi}\right)}_{=:H^\star_{t+1}}, \\ K_{t+1} &= e^{\omega_{t+1}b_{t+1}} \underbrace{\left((1-\delta_k)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t\right)}_{=:K^\star_{t+1}}, \text{where } \Phi(x) := \frac{\varphi_1}{1-\kappa}x^{1-\kappa} + \varphi_2. \end{aligned}$$

Finally, the disaster size $1-e^{\omega_{t+1}}$ also evolves stochastically according to

$$\begin{aligned} \omega_t &:= \bar{\omega} e^{\hat{\omega}_t}, \\ \hat{\omega}_{t+1} &= \rho_\omega \hat{\omega}_t + \epsilon_{\omega,t+1},\ \epsilon_{\omega,t} \sim \text{iidN}\left(0,\sigma_\omega^2\right), \end{aligned} \tag{3.4.2}$$

where $\bar{\omega} < 0$. We slightly deviate from the treatment in Gourio (2012) in the specification of the process governing the disaster size and allow autocorrelation but restrict outcomes to $\omega_t < 0$ so that disasters always have negative effects. The specification is similar to Fernández-Villaverde and Levintal (2018).[13]

Summing up, the household's problem in a centralized economy reads as follows

$$\begin{aligned} \max_{C_t,I_t,D_t,N_t,K^*_{t+1},H^*_{t+1}} / \min \quad & V_t = (1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}, \\ \text{s.t.} \quad & Y_t = K_t^\alpha (A_t N_t)^{1-\alpha}, \\ & Y_t = C_t + I_t + D_t, \\ & H_{t+1} = e^{\omega_{t+1}b_{t+1}(1-\phi)}\left(D_t^{1-\phi} + (1-\delta_h)H_t\right), \\ & K_{t+1} = e^{\omega_{t+1}b_{t+1}}\left((1-\delta_k)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t\right), \\ & D_t, I_t \geq 0, \end{aligned} \tag{3.4.3}$$

where K^*_{t+1} and H^*_{t+1} is the size of the stocks before b_{t+1} realizes and Φ remains defined as before. We define GDP again as the sum of consumption, both investment types and the implicit rent from housing.

Return Rates and Leverage The return rates on equity, housing and total risk are defined by (see also Gourio (2012) and Heiberger (2018)):

$$1 + R_{E,t+1} = e^{b_{t+1}\omega_{t+1}} \frac{r_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}(1-\delta_k + \Phi(\frac{I_{t+1}}{K_{t+1}}))}{q_t} = \frac{Y_{t+1} - W_{t+1}N_{t+1} - I_{t+1} + q_{t+1}K^\star_{t+2}}{q_t K^\star_{t+1}},$$

[13] Gourio (2012) additionally considers a transitory component of disasters. We checked the effects of a transitory shock component as well. Since we find that the effects for our targets are marginal, we omit the transitory component for the sake of simplicity.

$$1+R_{H,t+1} = e^{(1-\phi)b_{t+1}\omega_{t+1}} \frac{MRS^{H,C}_{t+1} + (1-\delta_h)P_{ht+1}}{P_{ht}} = \frac{\frac{\mu_h}{\mu_c}C_{t+1} - P_{h,t+1}D^{1-\phi}_{t+1} + P_{h,t+1}H^{\star}_{t+2}}{P_{h,t}H^{\star}_{t+1}},$$

$$1+R_{T,t+1} = \frac{\frac{\mu_h}{\mu_c}C_{t+1} - P_{h,t+1}D^{1-\phi}_{t+1} + P_{h,t+1}H^{\star}_{t+2} + Y_{t+1} - W_{t+1}N_{t+1} - I_{t+1} + q_{t+1}K^{\star}_{t+2}}{P_{h,t}H^{\star}_{t+1} + q_tK^{\star}_{t+1}},$$

where $r_t = \alpha(Y_t/K_t), W_t = (1-\alpha)(Y_t/N_t), q_t = 1/\Phi'(I_t/K_t)$ and $MRS^{H,C}_t = (\mu_h/\mu_c)(C_t/H_t)$ are the marginal product of capital, the real wage rate, Tobin's q and the marginal rate of substitution between housing and consumption, respectively. Moreover, the risk-free rate satisfies

$$1+R_{f,t} = \frac{1}{\mathbb{E}_t M_{t+1}},$$

where M_{t+1} denotes the model's stochastic discount factor given by

$$M_{t+1} := \beta\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left(\frac{V_{t+1}}{(\mathbb{E}_t V^{1-\theta}_{t+1})^{1/(1-\theta)}}\right)^{-\theta} \quad \text{with} \quad \Lambda_t := \mu_c \frac{\tilde{C}_t^{1-\frac{1}{\psi}}}{C_t}.$$

However, $R_{f,t}$ is the return rate of a real risk-free government bond which, as already noted, does not have an equivalent empirical counterpart. We therefore follow Barro (2006) and Gourio (2012) and assume that bonds in the model may also default during disasters. More concretely, we consider government (gb), corporate (cb) and housing (hb) bonds which differ by their recovery rates $\Gamma_{x,t}$ during disasters, $x \in \{\text{gb}, \text{cb}, \text{hb}\}$. The price $Q^{(T_x)}_{x,t}$ of such a bond with bond specific maturity T_x then satisfies the recursion

$$Q^{(T_x)}_{x,t} = \mathbb{E}_t[M_{t+1}\big(1-b_{t+1}+b_{t+1}\Gamma_{x,t+1}\big)Q^{(T_x-1)}_{x,t+1}], \quad \text{where } Q^{(0)}_{x,t+1} \equiv 1.$$

The ex-post return rates from holding bonds with maturity T_x for one period are defined by

$$1+R^{(T_x)}_{x,t+1} := \frac{\big(1-b_{t+1}+b_{t+1}\Gamma_{x,t+1}\big)Q^{(T_x-1)}_{x,t+1}}{Q^{(T_x)}_{x,t}}.$$

We assume that the rates at which bonds default during disasters are coupled to the disaster size $1-e^{\omega_{t+1}}$ via constant fractions $\chi_x \in [0,1]$ so that

$$\Gamma_{x,t+1} = 1-\chi_x(1-e^{\omega_{t+1}}).$$

Finally, since the return on equity in the data is calculated from stock returns, it includes leverage. This does not hold for housing returns. To be in line with the data, we also consider leveraged return rates in the model. More precisely, we assume that in each period the constant fraction $m_{cb} \in [0,1)$ of the firm's capital stock is financed by debt through bonds which all have maturity T_{cb}. Since the Modigliani and Miller theorem holds, the leveraged return rate on equity and the leveraged return rate on total risk are then given by

$$R^{lev}_{E,t+1} := \frac{1}{1-m_{cb}}R_{E,t+1} - \frac{m_{cb}}{1-m_{cb}}R^{(T_{cb})}_{cb,t+1},$$

$$R^{lev}_{T,t+1} := \frac{P_{h,t}H^{*}_{t+1}}{P_{h,t}H^{\star}_{t+1}+q_tK^{\star}_{t+1}}R_{H,t+1} + \frac{q_tK^{\star}_{t+1}}{P_{h,t}H^{*}_{t+1}+q_tK^{\star}_{t+1}}R^{lev}_{E,t+1}.$$

Table 3.7: Calibration

Parameter	Value	Description
β	0.995	discount factor
ψ	2	elasticity of intertemporal substitution
γ	5.5	coefficient of relative risk aversion
$\mu_c^{*)}$	0.30	weight of consumption in composite good
$\mu_h^{*)}$	0.07	weight of housing in composite good
α	0.36	capital share in production
ϕ	0.30	share of land in housing
κ	0.80	elasticity of Tobin's q wrt. investment-capital ratio
δ_k	0.022	rate of capital depreciation
δ_h	0.009	rate of housing depreciation
$\ln a$	0.005	growth rate
ρ_z	0.00	autocorrelation of log technology shock
ρ_ω	0.00	autocorrelation of log disaster size
ρ_p	0.95	autocorrelation of log disaster probability
σ_z	0.01	conditional standard deviation of log technology shock
σ_ω	0.67	conditional standard deviation of log disaster size
$\sigma_p/\sqrt{1-\rho_p^2}$	2.5	unconditional standard deviation of log disaster probability
$\bar{\omega}$	-0.067	disaster size
$\bar{p}/\exp(\frac{\sigma_p^2}{2(1-\rho_p^2)})$	0.0079	mean disaster probability
q	0.93	probability for disaster persistence
χ_{gb}	0.20	default loss of government bonds as fraction of disaster size
χ_{cb}	0.38	default loss of corporate bonds as fraction of disaster size
T_{gb}	1	maturity of the government bond
T_{cb}	10	maturity of the corporate bond
m_{cb}	0.37	corporate's financial leverage

Notes: $^{*)}$ Endogenous by the model.

3.4.2 Calibration

Our analysis considers different variations of the model. We start with a variation of the model, named F, which excludes housing before introducing housing into the model in variation G. In models F and G disaster risk is time-varying in that both the probability p_t and the disaster size ω_t follow stochastic processes (3.4.1) and (3.4.2), respectively. Model H shuts down the stochastic effect for p_t and model I for ω_t while in model J both effects are shut down.

First, the share α of capital in production and the depreciation rates δ_k and δ_h of productive capital and housing remain the same as in our variation of the Jermann (1998) model with housing (model B). We increase the coefficient of RRA moderately to $\gamma = 5.5$ and set the now disentangled EIS to $\psi = 2$ following Gourio (2012). We maintain our strategy to set the elasticities in the composite good in such way that the household's expenditures on housing account to 19 percent of his total consumption expenditures and that the household works one third of his time endowment on balanced growth (Model D/E). Moreover, the average growth rate a of technology during normal times is also kept the same as before and corresponds to the value in Gourio (2012). We set $\rho_z = 0$ and $\sigma_z = 0.01$ so that during normal times the stochastic process governing technological progress is identical to the process for the permanent component of productivity in Gourio (2012). The share of land in new houses is set to the upper bound $\phi = 0.3$ from Fehrle (2019) in order to fit the model closer to the data.[14]

The calibration of the 'free' parameters β and κ and of the additional parameters from the introduction of rare disasters is guided by Gourio (2012) and Fernández-Villaverde and

[14]We present and discuss the results of model G with $\phi = 0.106$ in Appendix 3.B. Favilukis, Ludvigson, and Nieuwerburgh (2017) proceed similar by setting the land share equal to 0.1 and 0.25.

Levintal (2018) but moderately adjusted in order to fit the model closer to the data. More precisely, we set $\beta = 0.995$ and $\kappa = 0.8$. Further, we follow Gourio (2012) and assume an iid process for the disaster size, i.e. $\rho_\omega = 0$. We choose $\bar{\omega} = -0.067$ and $\sigma_\omega = 0.67$ which implies a mean disaster size of approximately 8 percent. In comparison, the mean disaster size of the transitory and permanent components combined is approximately 6 percent in Gourio (2012). For the probability to enter a disaster, we choose a moderately larger autocorrelation $\rho_p = 0.95$ and a moderately lower standard deviation $\sigma_p = 2.5\sqrt{1-\rho_p^2}$ instead of $\rho_p = 0.9$ and $\sigma_p = 2.8\sqrt{1-\rho_p^2}$ in Gourio (2012). Finally, $\bar{p}$ is set such way that the average probability of entering a disaster is 0.72 percent—the same value used by Gourio (2012)—while the persistence of disasters is pinned down to $q = 0.93$—moderately above the value of $q = 0.914$ employed by Gourio (2012).

Lastly, we make the following assumptions for asset prices. Consistent with Barro (2006) and Gourio (2012) the default loss of government bonds is 20 percent of the disaster size, i.e. $\chi_{gb} = 0.2$, while the default loss of corporate bonds is set to a higher value of $\chi_{cb} = 0.38$. We consider government bonds with maturity of one period since our empirical counterpart are bills, and the maturity of corporate bonds is set to $T_{cb} = 10$. Gourio (2012) reports financial leverage of approximately 30 percent in the data. However, he interprets leverage in a broader way, i.e. also as operating leverage, and therefore chooses a larger level of leverage of approximately 50 percent for the calibration of his model. Our value of $m_{cb} = 0.37$ lies in between.

3.4.3 Results

Table 3.8 presents the asset return statistics for unlevered equity, for housing and for a real risk-free bond while Table 3.9 shows the return rates for a government bond with partial default in disasters and for leveraged equity. The business cycle statistics are summarized in Table 3.10. Note that we follow Gourio (2012) and, except for row G*, report statistics which are computed from samples where no disasters appear.

First, comparing rows F and G reveals that the introduction of housing into the model has only negligible effects on the return rates of unlevered and leveraged equity. The model (G) can explain return rates on leveraged equity and on government bonds which are close to the data and the model can replicate an equity premium of 6.56 percent. In accordance to the data, the return on housing (4.35 percent) and the housing premium (3.00 percent) turn out smaller than the return on equity and the equity premium. Yet, they remain approximately 1.5 percentage points below the values found in the data. Nevertheless, the model can closely match the empirical total risk premium. The model can further generate a low volatility of government bonds and reproduces the empirically observed standard deviations of returns and premia on housing fairly well. The standard deviations of equity returns and premia in the model are less than half of their empirical counterparts. Risk premia in the model are moderately more volatile than the risky return rates. Although the return rates and volatilities differ between the two risky assets, their Sharpe ratios turn out almost identical at approximately 0.9 and also coincide with the Sharpe ratio of total risk. Hence, the model can closely replicate the Sharpe ratio of housing of approximately 1.01 in the data but fails for the Sharpe ratio of equity which is substantially lower at 0.36 in the data.

Turning to business cycle statistics, the volatility of GDP in the model is too small. However, the model is able to generate relative volatilities of business investments (2.78), residential investments (5.56) and house prices (1.67) which all fit the data. Moreover, the model also reproduces the empirically observed correlation between GDP and residential investments and between GDP and house prices. The model shows almost perfect positive correlations between residential investments and house prices and between residential investments and business investments—and both correlations are also positive in the data.

Table 3.8: Simulated returns, premiums and second moments II (no default, no leverage)

	R_E	R_H	R_T	R_f	EP	HP	TP	SR_E	SR_H	SR_T
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27	0.36	1.01	0.75
					Model					
F	4.6	–	–	0.88	3.70	–	–	0.90	–	–
G	4.88	4.35	4.64	1.13	3.72	3.19	3.49	0.88	0.94	0.91
G*	3.31	2.8	3.08	−0.15	3.46	2.95	3.24	0.62	0.60	0.62
H	5.34	4.91	5.15	−1.25	6.66	6.21	6.46	5.33	8.51	6.33
I	4.41	4.04	4.25	2.32	2.06	1.69	1.90	0.64	0.71	0.68
J	4.70	4.42	4.58	1.33	3.34	3.06	3.22	2.67	4.43	3.19
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_f)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$			
USA	16.71	3.78	6.90	2.31	16.47	4.41	7.00			
					Model					
F	3.42	–	–	1.61	4.11	–	–			
G	3.56	2.64	3.15	1.58	4.22	3.41	3.85			
G*	5.82	5.26	5.56	3.55	5.55	4.92	5.26			
H	1.25	0.73	1.03	0.07	1.25	0.73	1.02			
I	2.78	1.86	2.36	1.07	3.20	2.37	2.81			
J	1.25	0.70	1.01	0.07	1.25	0.69	1.01			

Notes: Mean percentage returns of equity (R_E), housing (R_H), total risk (R_T) and bills (R_f) as well as the equity (EP), housing (HP), and the total risk premium (TP). The corresponding standard deviations $\sigma(X)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). We employ projection methods and simulated time series with 100,000 periods. The sample does **not** include disasters except for row G*. **F**: Benchmark rare disaster. **G**: F + Housing (no disaster sample). **G***: F + Housing (disaster sample). **H**: G but constant disaster probability. **I**: G but constant disaster size. **J**: G but constant disaster probability and size.

Table 3.9: Simulated returns, premiums and second moments II (default and leverage)

	R_E^{lev}	R_H	R_T^{lev}	R_{gb}	EP^{lev}	HP	TP^{lev}	SR_E	SR_H	SR_T
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27	0.36	1.01	0.75
					Model					
F	7.64	–	–	1.07	6.52	–	–	0.92	–	–
G	7.93	4.35	6.34	1.31	6.56	3.00	4.98	0.90	0.91	0.91
G*	6.08	2.80	4.63	0.03	6.04	2.76	4.60	0.66	0.61	0.65
H	10.87	4.91	8.25	−0.72	11.65	5.66	9.02	5.80	7.75	6.22
I	6.05	4.04	5.17	2.42	3.57	1.59	2.70	0.66	0.69	0.67
J	7.51	4.42	6.17	1.59	5.86	2.80	4.53	2.93	4.06	3.15
	$\sigma(R_E^{lev})$	$\sigma(R_H)$	$\sigma(R_T^{lev})$	$\sigma(R_{gb})$	$\sigma(EP^{lev})$	$\sigma(HP)$	$\sigma(TP^{lev})$			
USA	16.71	3.78	6.90	2.31	16.47	4.41	7.00			
					Model					
F	6.47	–	–	1.43	7.12	–	–			
G	6.67	2.64	4.87	1.39	7.29	3.29	5.50			
G*	9.34	5.26	7.44	3.45	9.13	4.51	7.08			
H	2.01	0.73	1.45	0.07	2.01	0.73	1.45			
I	5.03	1.86	3.62	0.94	5.41	2.29	4.01			
J	2.00	0.70	1.44	0.07	2.00	0.69	1.44			

Notes: Mean percentage returns of leveraged equity (R_E^{lev}), housing (R_H), leveraged total risk (R_T^{lev}) and bills (R_{gb}), as well as the leveraged equity (EP^{lev}), housing (HP), and the leveraged total risk premium (TP^{lev}) and the corresponding standard deviations $\sigma(X)$. We employ projection methods and simulated time series with 100,000 periods. The sample does **not** include disasters except for row G*. **F**: Benchmark rare disaster. **G**: F + Housing (no disaster sample). **G***: F + Housing (disaster sample). **H**: G but constant disaster probability. **I**: G but constant disaster size. **J**: G but constant disaster probability and size.

Table 3.10: Simulated business cycle statistics II

	σ_{GDP}	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$r_{RESI}^{P_h}$	r_{RESI}^{BUSI}	r_{RESI}^{GDP}	$r_{GDP}^{P_h}$
				Data				
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.64
				Model				
F	0.90	2.76	–	–	–	–	–	–
G	0.93	2.78	5.56	1.67	1.00	0.99	0.66	0.66
G*	2.72	1.94	3.55	1.07	1.00	0.96	0.63	0.63
H	0.84	1.12	1.78	0.53	1.00	1.00	1.00	1.00
I	0.89	2.22	4.01	1.20	1.00	0.98	0.69	0.69
J	0.84	1.12	1.70	0.51	1.00	1.00	1.00	1.00

Notes: σ_x is the standard deviation of x, r_y^x the correlation between x and y. Business cycle statistics from HP-filtered (1600) times series. We employ projection methods. Business cycle statistics are the mean outcome of repeated simulations of hp-filtered (1600) times series with 180 periods each. The sample does **not** include disasters except for row G*. **F**: Benchmark rare disaster. **G**: F + Housing (no disaster sample). **G***: F + Housing (disaster sample). **H**: G but constant disaster probability. **I**: G but constant disaster size. **J**: G but constant disaster probability and size.

The moments discussed so far from simulations without disasters are driven only by the agents' expectations about disaster whereas the actual occurrence of disasters is shut off. Row G* of the tables shows the moments from simulations which include disasters. With disasters in the sample, the mean return rate of the government bond already falls close to zero. The return on leveraged equity declines by approximately 2 percentage points and the equity premium decreases by 0.5 percentage points towards its empirical target. The return on housing and the housing premium decrease slightly less by approximately 1.5 and 0.25 percentage points, respectively, and we can observe similar effects for the return on total risk. Of course, the most obvious effect of samples with disasters is on the variables' second moments. The sample with disasters helps to increase the volatilities of the risky assets but also implies a counterfactual large standard deviation of the government bond. Moreover, GDP becomes too volatile and the model's fit of the relative standard deviations deteriorates.

The effects of time-varying disaster risk are illustrated in rows H, I, and J, which show the results from the model if the stochastic nature of the disaster probability, of the disaster size, or of both components is shut down. First, row H reveals that a time-varying probability for the economy to be hit by a disaster is essential for the model's dynamics. While the unlevered return rates on the risky assets and on total risk change only moderately if the probability for disasters is held constant, the risk-free rate and the return on bonds—government and corporate—decrease considerably. As a consequence of decreasing return rates on corporate bonds, the return on leveraged equity increases substantially by more than 3 percentage points and the leveraged equity premium now exceeds its empirical value almost by a factor of 2. On the other hand, since the housing premium is unlevered, it is only affected by the decreasing return on government bonds and, hence, rises only moderately above its empirical counterpart. The premium on total risk remains close to the average of the two risky assets. Further, row H shows that a time-varying probability to enter disasters is also the main factor to generate fluctuations in the return rates. In fact, with constant disaster probability the standard deviation of bonds falls close to zero and the standard deviations of returns on the risky assets and on total risk collapse by a factor of 4-5. Similarly, row H of Table 3.10 also illustrates that a time-varying disaster probability helps the model to generate the relative volatilities of business investments, residential investments and house prices and also helps to disentangle the otherwise perfect correlations between variables.

Since we assumed an uncorrelated shock process ($\rho_\omega = 0$), the model's results depend far less on the stochastic nature of the disaster size. In fact, shocks to the disaster size do not provoke any reactions of the model's variables in non-disaster periods but the effects show only indirect through expectations in the model solution. We can see from row I of tables 3.8 and 3.9 that, if the stochastic effect on the disaster size is shut off, the return rates on the risky assets and on total risk decline while the return on bonds increases. Moreover, a time varying disaster size helps to moderately increase the volatilities of all return rates and also helps to increase the relative volatilities of business investments, residential investments and house prices (see row I of Table 3.10). On the other hand, the variables' correlations remain almost unchanged.

3.4.4 Summary and discussion:

Figure 3.1 displays the reaction of the model's variables in response to a one time shock to technology z_t (panels (a)-(c)), to a one time shock to the probability p_t of entering a disaster (panels ((d)-(f)) and to a disaster which lasts for 5 periods (panels (g)-(i)) starting from the stochastic steady state in a non-disaster period.[15] We show percent deviations from the initial balanced growth path.

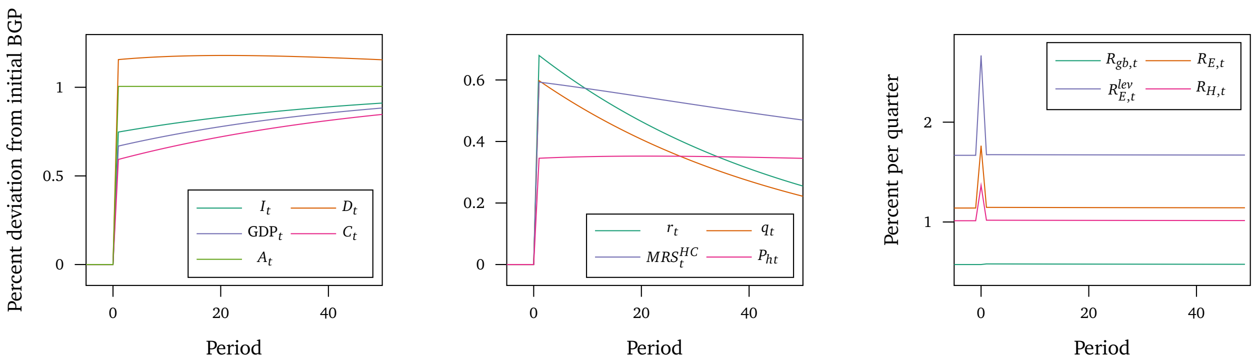
First, the variables' response to a 'classic' technology shock (panels (a)-(c)) is standard, and business investments, residential investments, and consumption increase in the period the shock hits the economy. An increase of business investments implies an increasing Tobin's q, $q_t = (1/\varphi_1)(I_t/K_t)^\kappa$, and increasing residential investments imply increasing house prices, $P_{h,t} = (1/(1-\phi))D_t^\phi$. Although D_t increases more than I_t, the elasticity κ of Tobin's q exceeds the elasticity ϕ of house prices and Tobin's q expands significantly more than house prices. Moreover, increasing productivity yields an increasing marginal product of capital and increasing consumption implies an increasing marginal rate of substitution between housing and consumption. In consequence, the returns on unlevered equity and on housing increase but—mainly due to the larger elasticity of Tobin's q—the return on unlevered equity dominates. Bonds do not react since the technology process is uncorrelated ($\rho_z = 0$), and debt additionally multiplies the effect for the leveraged return on equity.

On the other hand, an increase of the probability for the economy to enter a disaster has the following effects (see panels (d)-(f)). Positive autocorrelation ($\rho_p > 0$) implies an increased risk for a drop in productivity and for destruction of capital in the next period. In consequence, the representative agent lowers investments in productive capital and in residential structures and increases consumption instead. Decreasing investments entail drops in Tobin's q and in house prices. Although investments in residential structures decline more than investments in productive capital, the different elasticities again imply that the effect on Tobin's q dominates the effect on house prices. Moreover, a reduction of working hours implies a decreasing marginal product of capital r_t whereas increasing consumption implies that the marginal rate of substitution between housing and consumption increases. The more pronounced drop in Tobin's q compared to the drop in house prices combined with an increasing $MRS^{H,C}$ yields a larger contraction of the return on unlevered equity than of the return on housing and the effect is further amplified by leverage. Finally, increased disaster risk increases the stochastic discount factor so that bond prices increase. Yet, the effect is significantly smaller than on the risky assets.

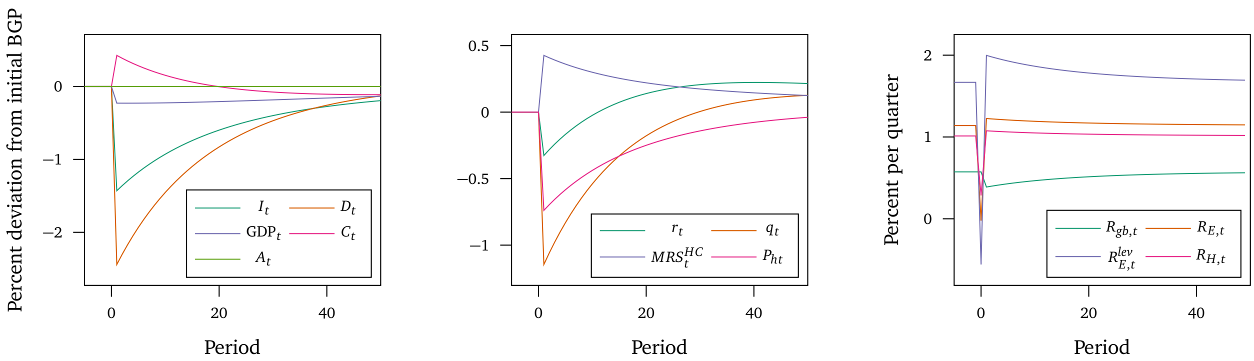
Lastly, an occurrence of a disaster (panels ((g)-(i)) implies that technology A_t drops by the factor $e^{\bar{\omega}}$ as long as the disaster continues. In the period the disaster starts, a second effect appears. The probability that the disaster remains persistent raises to $q = 0.93$ whereas the probability to enter a disaster was initially only $p \approx 0.0072$. The massive increase in probability for continued destruction of technology, capital and residential structures in the subsequent period has the previously described effects—amplified by a multitude. The two effects combined—drop in productivity and increased risk for the disaster to persist—cause huge drops of business

[15] As already noted, the assumption of an iid process for the disaster size ($\rho_\omega = 0$) implies that shocks to the disaster size do not provoke any reaction of the model's remaining variables.

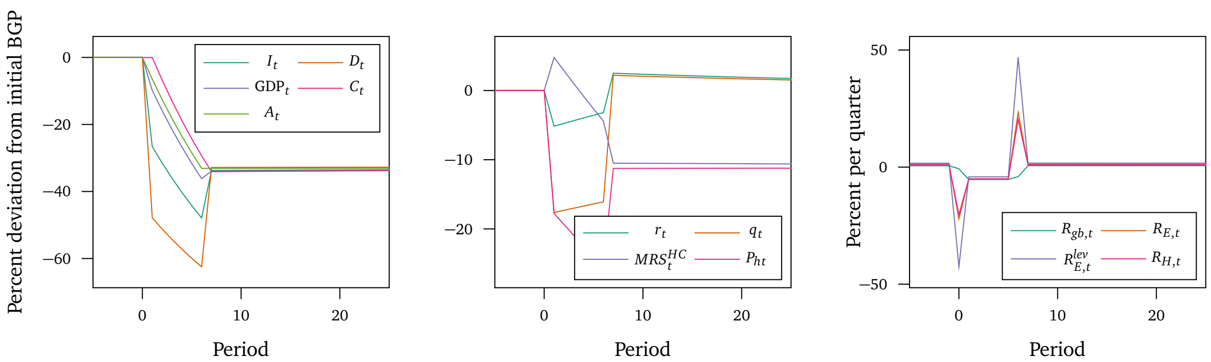
Figure 3.1: Impulse response functions to productivity and disaster risk shocks



(a) Technology shock to z_t (I) (b) Technology shock to z_t (II) (c) Technology shock to z_t (III)



(d) Disaster risk shock to p_t (I) (e) Disaster risk shock to p_t (II) (f) Disaster risk shock to p_t (III)



(g) Disaster $b_t = 1$ (I) (h) Disaster $b_t = 1$ (II) (i) Disaster $b_t = 1$ (III) investments and residential investments in the initial period of the disaster. In the following disaster periods, expectations do not change anymore until the disaster ends so that investments are only effected by decreasing technology, capital and residential structures. The initial drop of business investments exceeds the destruction of productive capital so that Tobin's q also collapses. In the following periods the effect turns and business investments decline by less than the rate at which capital is destructed so that Tobin's q begins to slowly recover. On the other hand, since land is not destructed, house prices continue to decline as long as D_t declines. Finally, once the disaster ends, the probability for the economy to be hit by a disaster again jumps back to $p \approx 0.0072$. The massive change in expectations leads to a boom immediately after the disaster. Both investments increase and so do Tobin's q and house prices. The huge drops in Tobin's q and in house prices at the start of the disaster yield huge drops in the return rates while the boom after the disaster ends implies huge yields of both risky assets.

Summing up, the model can generate different premia for unlevered equity and housing mainly due to different elasticities for Tobin's q and for house prices. Additionally, the gap between the two risky assets can be enlarged by leverage. However, we could not achieve further improvements of the model fit, in particular for the volatility of the return on equity, by fine-tuning the parameters controlling the elasticities of Tobin's q and of house prices. Increasing the elasticity κ of Tobin's q has the desired effect and helps to generate more volatile return rates on equity. However, it also implies a too large premium on equity compared to housing and, counterfactual to the data, reduces the volatility of business investments. Decreasing the elasticity of house prices ϕ also impairs the model's fit. At odds to the data, the return on housing and its volatility decrease while the Sharpe ratio of housing further increases.[16]

The main shortcoming of the model's asset price statistics, independent of the calibration, remains the fact that the Sharpe ratios of the two risky assets turn out far too similar.[17] In order to identify the reasons for this failure, we can return to equation (3.3.4). Note that by definition all bonds, $x \in \{\text{gb}, \text{cb}, \text{hb}\}$, also satisfy

$$\mathbb{E}_t\left[M_{t+1}R^{(T_x)}_{x,t+1}\right] = 1.$$

Moreover, the condition also holds for the levered return on equity

$$\mathbb{E}_t\left[M_{t+1}R^{lev}_{E,t+1}\right] = \frac{1}{1-m_{cb}}\mathbb{E}_t\left[M_{t+1}R_{E,t+1}\right] - \frac{m_{cb}}{1-m_{cb}}\mathbb{E}_t\left[M_{t+1}R^{(T_{cb})}_{cb,t+1}\right] = \frac{1-m_{cb}}{1-m_{cb}} = 1.$$

Hence, proceeding in an analogous way as before, the Sharpe ratio of housing and of leveraged equity can again be decomposed as

$$\begin{aligned} \mathrm{SR}^{lev}_E &:= \frac{\mathbb{E}\left[R^{lev}_{E,t+1} - R^{(T_{gb})}_{gb,t+1}\right]}{\sqrt{\mathrm{Var}\left[R^{lev}_{E,t+1} - R^{(T_{gb})}_{gb,t+1}\right]}} = -\frac{\sqrt{\mathrm{Var}\left[M_{t+1}\right]}}{\mathbb{E}\left[M_{t+1}\right]}\mathrm{Corr}\left[M_{t+1}, R^{lev}_{E,t+1} - R^{(T_{gb})}_{gb,t+1}\right], \\ \mathrm{SR}_H &:= \frac{\mathbb{E}\left[R_{H,t+1} - R^{(T_{gb})}_{gb,t+1}\right]}{\sqrt{\mathrm{Var}\left[R_{H,t+1} - R^{(T_{gb})}_{gb,t+1}\right]}} = -\frac{\sqrt{\mathrm{Var}\left[M_{t+1}\right]}}{\mathbb{E}\left[M_{t+1}\right]}\mathrm{Corr}\left[M_{t+1}, R_{H,t+1} - R^{(T_{gb})}_{gb,t+1}\right]. \end{aligned} \tag{3.4.4}$$

We summarize the decomposition of the Sharpe ratios in Table 3.11. Note however, that

[16] See Appendix 3.B.

[17] Different from the non-disaster models in section 3.3, we did not optimize the model's fit by matching moments with regard to the 'free' parameters. First, even with more efficient methods, as e.g. Polynomial Chaos Expansions proposed by Fehrle, Heiberger, and Huber (2019), such parameter inference would still be too time consuming. Second, we argue that it is the model's structure which is too simple to disentangle the Sharpe ratios of equity and housing.

Table 3.11: Risk premia components

	$\frac{2\sqrt{\mathrm{Var}[M_{t+1}]}}{(\mathbb{E}[M_{t+1}])^4}$	SR_E	SR_E^{lev}	SR_H	$r_{M,\mathrm{EP}}$	$r_{M,\mathrm{EP}^{lev}}$	$r_{M,\mathrm{HP}}$	$r_{\mathrm{EP},\mathrm{HP}}$	$r_{\mathrm{EP}^{lev},\mathrm{HP}}$
			Data						
USA	–	–	0.36	1.01	–	–	–	–	0.19
			Model						
G	0.64	0.88	0.90	0.91	−0.70	−0.76	−0.66	0.99	0.98
G*	0.96	0.62	0.66	0.61	−0.62	−0.66	−0.61	0.99	0.98

Notes: SR_E : annualized Sharpe ratio of equity, SR_H : annualized Sharpe ratio of housing, $r_{X,Y}$: correlation between variables X and Y where M_{t+1} : stochastic discount factor, $EP_{t+1} := R_{E,t+1} - R_{f,t}$: ex-post equity premium, $HP_{t+1} := R_{H,t+1} - R_{f,t}$: ex-post housing premium.

we compute moments from the simulation of the model's equilibrium outcomes so that the decompositions (3.4.4) only hold for samples that are consistent with the agent's expectations in the model solution, i.e. for samples which include disasters (G*). For non-disaster samples (G), it can be interpreted at best as a rough approximation which neglects the effects from the occurrence of disasters.

Nonetheless, the major deficit of the model is obvious. The return rates and also the premia between the two risky assets are again almost perfectly correlated so that their correlations with the stochastic discount factor are practically identical. The model's relatively simple structure implies that the effects of shocks on risky return rates are aligned and may only differ in size. This fact also becomes clearly evident from Figure 3.1 and the above interpretation of the impulse response functions. Hence, by (3.4.4) the Sharpe ratios of the two assets must be the same. Compared to the models without disaster risk in Table 3.6, the introduction of disasters risk raises the standard deviation of the stochastic discount factor by a factor of 4. The model can therefore explain substantially larger Sharpe ratios and matches the value of housing from the data. Yet, it now fails to simultaneously generate the lower Sharpe ratio of equity and, in consequence, produces far to involatile return rates on equity. Finally, the perfect correlation between the risky assets still implies that the mean *and* the standard deviation of returns on total risk are the weighted averages of the two risky assets. In order to explain the different Sharpe ratios of the two assets and in order to prevent the counterfactual characteristics of the total portfolio, it would be necessary to introduce effects into the model which help to dissolve the perfect correlation between the risky return rates.

Any mechanism that increases the volatility of the retrun on equity and decreases, in absolute terms, the correlation of the return on equity with the stochastic discount factor would improve the models fit. Assuming additionally that cooperate bonds could default in normal times meets these requirements.[18] The additional source of uncertainty increases the volatility of the return on equity while the assumption of independence decrease in absolute terms the correlation between the stochastic discount factor and the return on equity.

Other mechanisms concerning housing specific characteristics could improve the model's fit in general. E.g. due to the poor divisibility of housing, there may be credit constrained households which can only invest in equity. For them it would be impossible to smooth the consumption bundle by adjusting consumption and residential investment and subsequently the equity premia would increase. Albeit, housing investment participation distributes far broader and is less concentrated towards the top quantiles than the participation at the stock market as Kuhn, Schularick, and Steins (forthcomming) show. This indicates that the effect is minimal at best.

Among others, Mian, Rao, and Sufi (2013) find a large effect of housing wealth on consumption.

[18] Gourio (2012) argues e.g. the financial crises 2008 was not a great diasaster and US-treasury bonds and bills did not default. Nevertheless, a lot of cooporate bonds defaulted.

Modeling such a channel would increase in absolute terms the correlation between house prices and the stochastic discount factor and thus between the return on housing and the stochastic discount factor, which would separate the Sharpe ratios. Theoretical foundations for a large causal effect are given e.g. by Berger, Guerrieri, Lorenzoni, and Vavra (2017) and Guerrieri and Lorenzoni (2017). Gertler and Gilchrist (2018) summarize this channel in a review as follows: Mortgages are the household's most common structure of debt. Hence, declining house prices increase the households leverage ratio and the resulting tightened budget constraint forces the household to reduce his consumption spending.

Last, the fact that the risk of housing wealth is more idiosyncratic increases the volatility of the return on housing on an individual level. This is neither observable in a representative agent framework nor in the aggregated data of JKKST and thus explains potentially the difference in the Sharpe ratios. The PSID-based data from Flavin and Yamashita (2002) imply a Sharpe ratio of equity of 0.35, similar to the aggregated one, but the Sharpe ratio of housing is reduced by half to 0.47. Even if this explains a large part, a differential of one third remains.

3.5 Conclusion

In the present paper, we study the effects of housing on asset pricing statistics, especially on risk premia, in production economies. The stylized facts for asset prices which we consider are: i) a risk-free rate in the range of 1-2.2 percent together with a low volatility, ii) return rates on equity moderately larger than returns on housing, iii) premia on risky returns over 3 percent, iv) return rates and premia on equity which are at least twice as volatile as return rates and premia on housing and on total risk, and v) a Sharpe ratio of housing significantly larger than the Sharpe ratio of equity and similar to the Sharpe ratio of total risk. Since we study production economies, we also check the model's compatibility to the following well-established stylized facts of housing and business cycles: i) volatility of residential investments exceeds the volatility of business investments, ii) house prices are at least twice as volatile as GDP and are positively correlated with GDP, and iii) residential investments co-moves with house prices, GDP, and business investments.

We first introduce housing into non-disaster economies with habits and capital adjustment costs a la Jermann (1998). Housing provides the household with an insurance against fluctuations in the composite good and the model's ability to generate sizeable risk premia vanishes in consequence. However, the household's desire to smooth his consumption of the composite good induces demand effects which coincide with business cycle characteristics. Limitation of the household's option to smooth consumption of the composite good helps to generate modest risk premia but also eliminates the demand effects and hence reduces the model's fit to business cycle statistics. Moreover, the risk-free rate is far too volatile in the model and the model fails to explain the empirically observed differences between the returns on equity and on housing.

Second, we extend a standard RBC model with disaster risk similar to Gourio (2012) and Fernández-Villaverde and Levintal (2018) by housing. We find that the model can reproduce return rates on leveraged equity, on housing and on government bonds which are all close to the data. Moreover, the model can also match the volatility of government bonds and housing returns but equity returns are too involatile compared to the data. Different premia and different volatilities for equity and housing in the model are the result of i) different adjustment costs for productive capital and for residential structures which result in different elasticities of stock prices and of ii) leverage on equity. However, despite different premia and volatilities between the two risky assets, the model does not allow to disentangle the Sharpe ratios. Our calibration allows close replication of the empirical Sharpe ratios of housing and of total risk while the Sharpe ratio of equity exceeds its empirical counterpart substantially. Finally, regardless of its rather simple structure, the model is also able to generate relative volatilities of business investments, residential investments and house prices which all fit the data. Moreover, the model also reproduces the empirically observed correlation between GDP and residential investments and between GDP and house prices.

Bibliography

Abel, A. B. (1990): "Asset Prices under Habit Formation and Catching Up with the Joneses," in *American Economic Review, Papers and Proceedings*, 80(2), 38-42.

Bansal, R., and A. Yaron (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *The Journal of Finance*, 59(4), 1481–1509.

Barro, R. J. (2006): "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics*, 121(3), 823–866.

Berger, D., V. Guerrieri, G. Lorenzoni, and J. Vavra (2017): "House Prices and Consumer Spending," *The Review of Economic Studies*, 85(3), 1502–1542.

Boldrin, M., L. J. Christiano, and J. D. M. Fisher (2001): "Habit Persistence, Asset Returns, and the Business Cycle," *American Economic Review*, 91(1), 149–166.

Caldara, D., J. Fernández-Villaverde, J. F. Rubio-Ramírez, and W. Yao (2012): "Computing DSGE models with recursive preferences and stochastic volatility," *Review of Economic Dynamics*, 15(2), 188–206.

Campbell, J. Y., and J. H. Cochrane (1999): "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107(2), 205–251.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of political Economy*, 113(1), 1–45.

Cochrane, J. H., and L. P. Hansen (1992): "Asset Pricing Explorations for Macroeconomics," *NBER Macroeconomics Annual*, 7, 115–165.

Davis, M. A., and J. Heathcote (2005): "Housing and the Business Cycle," *International Economic Review*, 46(3), 751–784.

——— (2007): "The price and quantity of residential land in the United States," *Journal of Monetary Economics*, 54(8), 2595 – 2620.

Dorofeenko, V., G. S. Lee, and K. D. Salyer (2014): "Risk shocks and housing supply: A quantitative analysis," *Journal of Economic Dynamics and Control*, 45, 194–219.

Epstein, L. G., and S. E. Zin (1989): "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–969.

Favilukis, J., S. C. Ludvigson, and S. V. Nieuwerburgh (2017): "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk Sharing in General Equilibrium," *Journal of Political Economy*, 125(1), 140–223.

Fehrle, D. (2019): "Housing and the business cycle revisited," *Journal of Economic Dynamics and Control*, 99, 103–115.

FEHRLE, D., C. HEIBERGER, AND J. HUBER (2019): "Polynomial Chaos Expansion for efficient DSGE Model estimation and evaluation," unpublished memo, University of Augsburg.

FERNÁNDEZ-VILLAVERDE, J., AND O. LEVINTAL (2018): "Solution methods for models with rare disasters," *Quantitative Economics*, 9(2), 903–944.

FLAVIN, M., AND T. YAMASHITA (2002): "Owner-occupied housing and the composition of the household portfolio," *American Economic Review*, 92(1), 345–362.

GERTLER, M., AND S. GILCHRIST (2018): "What Happened: Financial Factors in the Great Recession," *Journal of Economic Perspectives*, 32(3), 3–30.

GOURIO, F. (2012): "Disaster Risk and Business Cycles," *American Economic Review*, 102(6), 2734–66.

GROSSMANN, V., B. LARIN, H. T. LÖFFLAD, AND T. STEGER (2019): "Distributional effects of surging housing costs under Schwabe's Law," Discussion paper.

GUERRIERI, V., AND G. LORENZONI (2017): "Credit Crises, Precautionary Savings, and the Liquidity Trap," *The Quarterly Journal of Economics*, 132(3), 1427–1467.

HEER, B., AND A. MAUSSNER (2009): *Dynamic General Equilibrium Modeling*. Springer Berlin Heidelberg.

HEIBERGER, C. (2018): "Asset Prices, Epstein Zin Utility, and Endogenous Economic Disasters," doctoral thesis, Universität Augsburg.

HEIBERGER, C., AND H. RUF (2019): "Epstein-Zin Utility, Asset Prices, and the Business Cycle Revisited," *German Economic Review*, 20(4), 730–758.

JACCARD, I. (2011): "Asset pricing and housing supply in a production economy," *The BE Journal of Macroeconomics*, 11(1), 1–40.

JAIMOVICH, N., AND S. REBELO (2009): "Can News about the Future Drive the Business Cycle?," *American Economic Review*, 99(4), 1097–1118.

JERMANN, U. J. (1998): "Asset pricing in production economies," *Journal of Monetary Economics*, 41, 257 –275.

JORDÀ, Ò., K. KNOLL, D. KUVSHINOV, M. SCHULARICK, AND A. M. TAYLOR (2019): "The Rate of Return on Everything, 1870 – 2015," *The Quarterly Journal of Economics*, 134(3), 1225–1298.

JORDÀ, Ò., M. SCHULARICK, AND A. M. TAYLOR (2019): "The Total Risk Premium Puzzle," *NBER Working Paper*, 25653, 1–56.

JUDD, K. L., L. MALIAR, S. MALIAR, AND R. VALERO (2014): "Smolyak method for solving dynamic economic models: Lagrange interpolation, anisotropic grid and adaptive domain," *Journal of Economic Dynamics and Control*, 44(C), 92–123.

KNOLL, K., M. SCHULARICK, AND T. STEGER (2017): "No Price Like Home: Global House Prices, 1870-2012," *American Economic Review*, 107(2), 331–353.

KUHN, M., M. SCHULARICK, AND U. I. STEINS (forthcomming): "Income and Wealth Inequality in America, 1949–2016," *Journal of Political Economy*.

KYDLAND, F. E., P. RUPERT, AND R. SUSTEK (2016): "Housing Dynamics over the Business Cycle," *International Economic Review*, 57(4), 1149–1177.

LANDVOIGT, T., M. PIAZZESI, AND M. SCHNEIDER (2015): "The Housing Market(s) of San Diego," *American Economic Review*, 105(4), 1371–1407.

LUCAS, R. E. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46(6), 1429–1445.

MEHRA, R., AND E. C. PRESCOTT (1985): "The equity premium: A puzzle," *Journal of Monetary Economics*, 15(2), 145–161.

MIAN, A., K. RAO, AND A. SUFI (2013): "Household Balance Sheets, Consumption, and the Economic Slump," *The Quarterly Journal of Economics*, 128(4), 1687–1726.

NGUYEN, Q. H. (2018): "Revisiting housing and the business cycle," *Journal of Housing Economics*, 41, 85–92.

PIAZZESI, M., M. SCHNEIDER, AND S. TUZEL (2007): "Housing, consumption and asset pricing," *Journal of Financial Economics*, 83(3), 531–569.

RIETZ, T. A. (1988): "The equity risk premium a solution," *Journal of Monetary Economics*, 22(1), 117–131.

SWANSON, E. T. (2012): "Risk Aversion and the Labor Margin in Dynamic Equilibrium Models," *American Economic Review*, 102(4), 1663–91.

UHLIG, H. (2007): "Explaining Asset Prices with External Habits and Wage Rigidities in a DSGE Model.," *The American Economic Review, Papers and Proceedings*, 97(2), 239 – 243.

WEIL, P. (1989): "The equity premium puzzle and the risk-free rate puzzle," *Journal of Monetary Economics*, 24(3), 401–421.

Appendix

3.A Stylized facts and data Resources

3.A.1 Stylized facts

In Table 3.A1 we present return rates for all countries from the JKKST database. We observe the following stylized facts. First, risk premia in all countries are sizeable with equity premia between 1.17 percent in Portugal and 12.91 percent in Finland, and housing premia between 3.47 and 8.39 percent in Germany and Norway, respectively. Second, in all countries except for Italy and Portugal the return on housing is lower than the return on equity. Third, in all countries listed the volatility of returns and premia on equity exceeds the volatility of returns and premia on housing and the volatility of the risk-free rate is the smallest. Fourth, the Sharp ratio of housing is larger than of equity in all listed countries. Last, there is no systematic nexus between the return on housing and on equity.

Table 3.A2 displays the business cycle statistics for the same countries. Note that for several continental European countries the standard deviation of residential investment exceeds the standard deviation of business investment only slightly or is even smaller. More precisely, this is the case for France, Finland, Germany, Italy, the Netherlands, Norway, Portugal, and Spain. House prices are pro-cyclical with GDP and more volatile than GDP in all countries except for Germany. Business investment and residential investment are positively correlated in all countries but Sweden and Australia, and house prices and residential investment are positively correlated throughout.

3.A.2 Sources

The data pertaining the rates of return, mortgage etc. are from JKKST. The source of the data pertaining the business cycle statistics is listed below:

- GDP, residential investment, non-residential investment: OECD Economic Outlook Nov 2018; Denmark: Statistics Denmark.
- House prices: OECD Real house price indices, s.a. 16.05.2019 devided by the OECD Economic Outlook Nov 2018 CPI Deflator.
- Population: FRA, USA: OECD Total population PERSA: Persons, seasonally adjusted; UK: Office for National Statistics UK, resident population: mid-year estimates (Qtly data interpolated (by the Office); Otherwise: Yearly, Worldbank, midyear (interpolated (by own calculation)).
- Home ownership rates: Japan (2007): http://www.stat.go.jp/english/index.html; USA (2007), Australia (2003): https://www.oecd.org/eco/growth/evolution

  %20of%20homeownership%20rates.pdf p.212; Otherwise (2010): EUROSTAT "Eurostat - Data Explorer - Distribution of population by tenure status, type of household and income group"

Table 3.A1: Returns, premiums and second moments

	R_E	R_H	R_T	R_f	EP	HP	TP
AUS	6.84	7.01	7.22	2.00	4.84	5.02	5.22
BEL	10.29	8.02	8.65	2.49	7.80	5.53	6.16
DNK	13.35	6.6	8.69	0.72	12.63	5.88	7.96
FIN	14.17	8.84	11.67	1.25	12.91	7.59	10.41
FRA	9.61	5.78	6.61	2.24	7.37	3.54	4.37
GER	9.04	4.75	5.75	1.28	7.76	3.47	4.47
ITA	4.76	5.62	5.89	1.55	3.21	4.07	4.34
JPA	5.86	5.54	6.19	0.98	4.88	4.56	5.21
NLD	8.93	7.83	8.12	1.44	7.49	6.38	6.68
NOR	13.03	9.82	10.66	1.43	11.60	8.39	9.23
PRT	2.92	6.65	6.62	1.75	1.17	4.89	4.87
ESP	7.10	4.43	4.89	0.69	6.40	3.74	4.20
SWE	12.29	8.82	10.56	0.91	11.38	7.92	9.65
CH	7.23	6.01	6.98	−0.17	7.40	6.19	7.16
UK	8.00	7.00	7.47	1.56	6.44	5.44	5.91
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_f)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$
AUS	21.53	5.71	6.02	3.32	21.32	6.22	6.27
BEL	22.99	6.04	6.37	2.84	23.08	6.61	7.02
DNK	23.84	7.72	8.91	1.56	24.33	7.24	8.93
FIN	37.72	9.13	21.58	4.52	36.90	9.64	21.39
FRA	24.11	5.52	6.95	2.55	23.98	6.18	7.39
GER	22.82	3.12	5.09	1.73	23.12	4.30	5.99
ITA	27.98	10.77	10.07	3.18	27.57	11.51	10.74
JPA	20.15	6.53	8.10	2.53	19.94	6.47	8.03
NLD	22.06	9.14	9.40	2.91	22.13	9.68	9.91
NOR	29.60	8.51	9.02	2.39	29.66	9.64	10.12
PRT	26.86	7.26	7.64	2.57	26.81	7.39	7.64
ESP	27.13	8.36	8.62	4.43	25.93	8.19	8.10
SWE	26.03	7.35	11.67	2.06	25.75	7.22	11.42
CH	21.61	4.59	8.01	2.29	21.41	4.94	7.97
UK	8.00	7	7.47	1.56	6.44	5.44	5.91
USA	16.71	3.78	6.90	2.31	16.47	4.41	7.00
	SR_E	SR_H	SR_T	$r_{\text{EP,HP}}$	$r_{\text{EP,TP}}$	$r_{\text{HP,TP}}$	
				Data			
AUS	0.23	0.81	0.83	−0.09	0.58	0.72	
BEL	0.34	0.84	0.88	−0.05	0.43	0.87	
DNK	0.52	0.81	0.89	0.37	0.81	0.83	
FIN	0.35	0.79	0.49	0.25	0.83	0.5	
FRA	0.31	0.57	0.59	0.06	0.63	0.76	
GER	0.34	0.81	0.75	0.21	0.79	0.74	
ITA	0.12	0.35	0.4	−0.24	−0.01	0.97	
JPA	0.24	0.7	0.65	0.16	0.79	0.72	
NLD	0.34	0.66	0.67	0.02	0.52	0.84	
NOR	0.39	0.87	0.91	−0.01	0.46	0.86	
PRT	0.04	0.66	0.64	0.07	0.49	0.89	
ESP	0.25	0.46	0.52	0.09	0.4	0.94	
SWE	0.44	1.1	0.85	0.01	0.8	0.5	
CH	0.35	1.25	0.9	−0.19	0.88	0.18	
UK	0.27	0.61	0.69	−0.13	0.72	0.54	
USA	0.36	1.01	0.75	0.19	0.9	0.56	

Notes: Mean percentage returns of equity (R_E), housing (R_H), total risk (R_T) and bills (R_f) as well as the equity (EP), housing (HP), and the total risk premium (TP) and the corresponding standard deviations $\sigma(X)$. SR_E : Sharpe ratio of equity, SR_H : Sharpe ratio of housing, SR_T : Sharpe ratio of total risk $r_{X,Y}$: correlation between variables X and Y where $EP_{t+1} := R_{E,t+1} - R_{f,t}$: ex-post equity premium, $HP_{t+1} := R_{H,t+1} - R_{f,t}$: ex-post housing premium $TP_{t+1} := R_{T,t+1} - R_{f,t}$: ex-post total risk premium. Periods: Australia 1970-2015, Belgium 1976-2015, Denmark 1995-2015, Finland 1970-2015, France 1980-2015, Germany 1991-2015, Italy 1970-2015, Japan 1963-2015, the Netherlands 1970-2015, Norway 1978-2015, Portugal 1988-2015. Spain 1971-2015, Sweden 1970-2015, Switzerland 1970-2015, United Kingdom 1969-2015, USA 1970-2015, data from JKKST, own calculations.

Table 3.A2: Empirical business cycle statistics

	σ_{GDP}	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$r^{P_h}_{RESI}$	r^{BUSI}_{RESI}	r^{GDP}_{RESI}	$r^{P_h}_{GDP}$
AUS	1.24	3.77	6.65	3.56	0.55	−0.12	0.56	0.35
BEL	1.02	4.21	7.19	3.54	0.62	0.22	0.47	0.31
DNK	1.42	3.52	5.95	3.96	0.60	0.28	0.66	0.75
FIN	2.21	2.87	3.15	3.00	0.73	0.40	0.67	0.66
FRA	0.95	2.75	3.17	3.19	0.65	0.64	0.81	0.48
GER	1.47	2.54	2.20	0.82	0.06	0.49	0.57	−0.16
ITA	1.44	2.66	1.67	3.73	0.25	0.37	0.46	0.15
JPA	1.59	2.41	3.84	2.70	0.31	0.27	0.45	0.55
NLD	1.37	5.86	5.57	4.08	0.40	0.31	0.48	0.35
NOR	1.46	4.68	4.51	3.72	0.60	0.28	0.31	0.56
PRT	1.58	3.37	2.58	1.87	0.40	0.56	0.64	0.50
ESP	1.33	3.48	3.42	4.17	0.43	0.69	0.77	0.61
SWE	1.51	4.46	5.20	2.95	0.42	−0.22	0.04	0.57
CHE	1.64	–	–	2.69	–	–	–	0.61
UK	1.58	2.68	5.56	4.85	0.51	0.16	0.69	0.71
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.64

Notes: Business cycle statistics are from quarterly logged per capita hp-filtered (1600) data. σ_x is the standard deviation of x, r^x_y the correlation between x and y. RESI=residential investment, BUSI=non-residential investment, P_h house prices. Periods: Australia 1970-2015, Belgium 1976-2015, Denmark 1995-2015, Finland 1970-2015, France 1980-2015, Germany 1991-2015, Italy 1970-2015, Japan 1963-2015, the Netherlands 1970-2015, Norway 1978-2015, Portugal 1988-2015. Spain 1971-2015, Sweden 1970-2015, Switzerland 1970-2015, United Kingdom 1969-2015, USA 1970-2015, Data: See Appendix 3.A, own calculations.

3.B Further Results

Economies with Non-Disaster risk

Higher share of land's value in new houses: We show the results from increasing the share of land in the Jermann (1998) model with housing to $\phi = 0.3$ in row K of tables 3.B2 and 3.B3. First, the return rates on both risky assets increase moderately while the risk-free rate decreases slightly. Nevertheless, the model can still not produce sizable risk premia. The volatility of residential investment decreases only moderately and remains far too large. On the other hand, house prices in the model become noticeably more volatile than empirically observed.

We also present the results for the two-sector model a la Boldrin, Christiano, and Fisher (2001) with $\phi = 0.3$ in rows L, M, and N of tables 3.B2 and 3.B3. We fine-tune the free parameters by matching moments in the same way as we did for $\phi = 0.106$ and summarize the parameters' values in Table 3.B1. Risk premia decrease moderately in model L compared to model C and again vanish if labor supply is endogenously determined in model M. In model N with limited sectoral labor mobility changes are only negligible compared to model E. The volatility of house prices increases in all three models and, as Fehrle (2019) shows, the correlations also increase. Nevertheless, with labor market frictions house prices do not co-move with residential investment.

To sum up, an increase in the share of land's value in new houses does not affect the conclusion that the present framework can not simultaneously reproduce asset pricing statistics and business cycle statistics as observed in the data.

Habitat without habits: The effects of habits in consumption, housing and leisure, respectively, can be discussed by setting the corresponding habit parameter to zero in model E. First, rows N of tables 3.B2 and 3.B3 show that habits in housing have negligible consequences for the presented business cycle characteristics and only small significance for risk premia. Without habits in housing, risk premia on both assets are reduced by approximately 0.8 percentage points. Rows O of the same tables show that without habits in leisure risk premia are halved. Moreover,

Table 3.B1: Calibration

	L	M / N
$\beta^\star$	0.999	0.999
η	5	5
$\mu_c^{*)}$	0.72	0.49
$\mu_h^{*)}$	0.18	0.19
χ_c	0.825	0.825
χ_h	0.70	0.70
χ_n	–	0.95
a_y	1.005	1.005
a_d	1.002	1.002
ϕ	0.30	0.30
α_y	0.25	0.25
α_d	0.20	0.20
κ_y	6.25	6.25
κ_d	1.25	1.25
δ_k	0.022	0.022
δ_h	0.009	0.009
σ_y	0.0094	0.0094
ρ_y	0.966	0.966
σ_d	0.0172	0.0172
ρ_d	0.923	0.923

Notes: $^{*)}$ Endogenous by the model. **L**: C but $\phi = 0.3$. **M/N**: D/E but $\phi = 0.3$.

the volatility of house prices reduces towards its empirical counterpart but business investment becomes even less volatile and the correlation between GDP and residential investment also moves farther away from the value in the data.

Habits in consumption, see row P of tables 3.B2 and 3.B3, have the largest effect on the results from model E. Without habits in consumption the household's marginal utility does not fluctuate enough between different realizations of the productivity shock so that risk premia reduce drastically. While business investment becomes less volatile, the standard deviation of residential investment increases. Moreover, the correlation between house prices and residential investment becomes positive and the correlation between business and residential investment also increases substantially.

Housing with disaster risk

Lower share of land's value in new houses: Tables 3.B2 and 3.B3 display in rows R and R^{lev} the effects from lowering the share of land in our disaster economies to $\phi = 0.106$ as in Davis and Heathcote (2005). We find that returns on equity do not change by much but housing premia decrease by 0.7 percentage-points and the standard deviation of returns on housing is reduced by 0.96. The Sharpe ratio of housing exceeds its empirical counterpart. Moreover, the volatility of house prices drops below the standard deviation of GDP. The lower share of land in new houses increases the volatility of residential investment which now becomes too large.

Table 3.B2: Returns, premiums and second moments

	R_E	R_H	R_T	R_f	EP	HP	TP
USA	7.45	6.01	6.84	1.57	5.88	4.45	5.27
				Model			
K	4.83	4.59	4.72	4.03	0.77	0.54	0.67
L	4.84	4.61	4.70	0.50	4.33	4.10	4.19
M	2.70	2.60	2.64	1.95	0.74	0.64	0.68
N	4.92	4.69	4.78	1.39	3.49	3.27	3.36
O	4.31	4.11	4.19	1.67	2.61	2.41	2.49
P	3.40	3.32	3.36	2.05	1.34	1.26	1.29
Q	2.66	2.50	2.57	2.02	0.63	0.47	0.54
R	5.16	3.87	4.59	1.36	3.76	2.49	3.20
R^{lev}	8.24	3.87	6.30	1.55	6.62	2.30	4.70
	$\sigma(R_E)$	$\sigma(R_H)$	$\sigma(R_T)$	$\sigma(R_f)$	$\sigma(EP)$	$\sigma(HP)$	$\sigma(TP)$
USA	16.71	3.78	6.9	2.31	16.47	4.41	7.00
				Model			
K	8.25	6.01	7.29	1.45	8.09	5.80	7.11
L	22.43	21.47	21.86	8.49	20.62	19.57	19.99
M	8.82	7.70	8.14	2.13	8.54	7.37	7.84
N	22.77	21.81	22.19	12.84	18.62	17.42	17.90
O	19.93	19.00	19.39	11.59	16.04	14.87	15.36
P	14.75	14.17	14.41	8.65	11.92	11.17	11.49
Q	8.32	6.63	7.32	3.05	7.73	5.86	6.64
R	3.71	1.68	2.73	1.51	4.33	2.59	3.45
R^{lev}	6.83	1.68	4.45	1.32	7.41	2.44	5.07

Notes: Mean percentage returns of equity (R_E), housing (R_H), total risk (R_T) and bills (R_f) as well as the equity (EP), housing (HP), and the total risk premium (TP) and the corresponding standard deviations $\sigma(X)$. We employ a second order perturbation and simulated time series with 100,000 periods. **K**: B but $\phi = 0.3$. **L**:**L**: C but $\phi = 0.3,, \chi_h = 0.7$. **M**: D but $\phi = 0.3\chi_h = 0.7$. **N**: E but $\phi = 0.3, \chi_h = 0.7$. **O**: E but $\chi_h = 0$. **P**: E but $\chi_n = 0$. **Q**: E but $\chi_c = 0$. **R**: G but $\phi = 0.106$. **R**^{lev}: R but with leverage and default, housing remain not leveraged.

Table 3.B3: Simulated business cycle statistics III

	σ_{GDP}	$\frac{\sigma_{BUSI}}{\sigma_{GDP}}$	$\frac{\sigma_{RESI}}{\sigma_{GDP}}$	$\frac{\sigma_{P_h}}{\sigma_{GDP}}$	$r^{P_h}_{RESI}$	r^{BUSI}_{RESI}	r^{GDP}_{RESI}	$r^{P_h}_{GDP}$
					Data			
USA	1.52	2.91	6.85	2.03	0.67	0.07	0.72	0.63
					Model			
K	1.05	1.10	11.44	3.26	0.99	0.98	0.98	0.98
L	2.11	1.05	1.03	4.79	−0.03	0.00	0.06	0.96
M	1.42	0.77	4.39	3.04	0.80	0.83	0.84	0.94
N	1.9	0.96	2.12	4.23	0.04	0.14	0.39	0.91
O	1.95	0.84	1.83	3.57	−0.06	0.07	0.35	0.88
P	1.89	0.65	2.16	2.73	−0.1	0.01	0.28	0.89
Q	1.44	0.71	2.95	2.17	0.11	0.35	0.52	0.91
R	1.01	2.59	8.51	0.90	1.00	0.89	0.68	0.68

Notes: σ_x is the standard deviation of x, r^x_y the correlation between x and y. Business cycle statistics from HP-filtered (1600) times series. We employ a second order perturbation and report the average outcomes from repeated simulations with 180 periods. **K**: B but $\phi = 0.3$. **L**: C but $\phi = 0.3,, \chi_h = 0.7$. **M**: D but $\phi = 0.3\chi_h = 0.7$. **N**: E but $\phi = 0.3, \chi_h = 0.7$. **O**: E but $\chi_h = 0$. **P**: E but $\chi_n = 0$. **Q**: E but $\chi_c = 0$. **R**: G but $\phi = 0.106$.

3.C Economies with non-disaster risk

We summarize the details for the models discussed in section 3.3.

3.C.1 Housing with Jermann (1998)

The equilibrium conditions from the optimization problem (3.3.1) for the benchmark Jermann (1998) model which is extended by housing are determined by

$$
\begin{aligned}
\Lambda_t &= \mu_c (C_t - C_{ht})^{\mu_c(1-\eta)-1} (H_t - H_{ht})^{\mu_h(1-\eta)}, && \text{(3.C.1a)}\\
Y_t &= a^{(1-\alpha_y)t} Z_t K_t^{\alpha_y}, && \text{(3.C.1b)}\\
Y_t &= C_t + I_t + D_t, && \text{(3.C.1c)}\\
r_t &= \alpha_y \frac{Y_t}{K_t}, && \text{(3.C.1d)}\\
P_{h,t} &= \frac{1}{1-\phi} D_t^{\phi}, && \text{(3.C.1e)}\\
q_t &= \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)}, && \text{(3.C.1f)}\\
K_{t+1} &= (1-\delta_k) K_t + \Phi\left(\frac{I_t}{K_t}\right) K_t, && \text{(3.C.1g)}\\
H_{t+1} &= (1-\delta_h) H_t + D_t^{1-\phi}, && \text{(3.C.1h)}\\
C_{h,t+1} &= \chi_c C_t, && \text{(3.C.1i)}\\
H_{h,t+1} &= \chi_h H_t, && \text{(3.C.1j)}\\
q_t &= \mathbb{E}_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}\left(r_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\left(1 - \delta_k + \Phi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right)\right], && \text{(3.C.1k)}\\
P_{h,t} &= \mathbb{E}_t\left[\beta \frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\mu_h}{\mu_c}\frac{C_{t+1} - C_{h,t+1}}{H_{t+1} - H_{h,t+1}} + P_{h,t+1}(1-\delta_h)\right)\right], && \text{(3.C.1l)}
\end{aligned}
$$

given the state variables $K_t, H_t, C_{h,t}, H_{h,t}$ and Z_t. Additionally, the log of productivity follows the exogenous AR(1)-process

$$\ln Z_{t+1} = \rho_y \ln Z_t + \epsilon_{t+1}, \; \epsilon_t \sim \text{iidN}(0, \sigma_y^2).$$

Finally, GDP is defined by

$$\text{GDP}_t = Y_t + MRS_t^{H,C} H_t, \;\text{ where } MRS_t^{H,C} = \frac{\mu_h}{\mu_c}\frac{C_t - C_{ht}}{H_t - H_{ht}}.$$

We re-scale the variables by $k_t := \frac{K_t}{a^t}$, $h_t := \frac{H_t}{a^{(1-\phi)t}}$, $c_{h,t} := \frac{C_{h,t}}{a^t}$, $h_{h,t} := \frac{H_{h,t}}{a^{(1-\phi)t}}$, $y_t := \frac{Y_t}{a^t}$, $c_t := \frac{C_t}{a^t}$, $i_t := \frac{I_t}{a^t}$, $d_t := \frac{D_t}{a^t}$, $p_{h,t} := \frac{P_{h,t}}{a^{\phi t}}$, and $\lambda_t := \frac{\Lambda_t}{a^{((\mu_c+(1-\phi)\mu_h)(1-\eta)-1)t}}$. Hence, system (3.C.1) in scaled variables reads

$$
\begin{aligned}
\lambda_t &= \mu_c (c_t - c_{ht})^{\mu_c(1-\eta)-1} (h_t - h_{ht})^{\mu_h(1-\eta)}, && \text{(3.C.2a)}\\
y_t &= Z_t k_t^{\alpha_y}, && \text{(3.C.2b)}\\
y_t &= c_t + i_t + d_t, && \text{(3.C.2c)}\\
r_t &= \alpha_y \frac{y_t}{k_t}, && \text{(3.C.2d)}
\end{aligned}
$$

$$p_{h,t} = \frac{1}{1-\phi} d_t^{\phi}, \tag{3.C.2e}$$

$$q_t = \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)}, \tag{3.C.2f}$$

$$ak_{t+1} = (1-\delta_k)k_t + \Phi\left(\frac{i_t}{k_t}\right)k_t, \tag{3.C.2g}$$

$$a^{1-\phi}h_{t+1} = (1-\delta_h)h_t + d_t^{1-\phi}, \tag{3.C.2h}$$

$$ac_{h,t+1} = \chi_c c_t, \tag{3.C.2i}$$

$$a^{1-\phi}h_{h,t+1} = \chi_h h_t, \tag{3.C.2j}$$

$$q_t = \mathbb{E}_t\left[\beta a^{(\mu_c+(1-\phi)\mu_h)(1-\eta)-1}\frac{\lambda_{t+1}}{\lambda_t}\left(r_{t+1} - \frac{I_{i+1}}{k_{t+1}} + q_{t+1}\left(1-\delta_k + \Phi\left(\frac{i_{t+1}}{k_{t+1}}\right)\right)\right)\right], \tag{3.C.2k}$$

$$p_{h,t} = \mathbb{E}_t\left[\beta a^{(\mu_c+(1-\phi)\mu_h)(1-\eta)-1+\phi}\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\mu_h}{\mu_c}\frac{c_{t+1}-c_{h,t+1}}{h_{t+1}-h_{h,t+1}} + p_{h,t+1}(1-\delta_h)\right)\right]. \tag{3.C.2l}$$

3.C.2 Moving to Boldrin, Christiano, and Fisher (2001)

We continue to present the details for the two-sector models from section 3.3.

Sectoral frictions with exogenous labor supply:

$$\begin{aligned}
\max_{C_t,D_t,I_{yt},I_{dt},K_{yt+1},K_{dt+1},H_{t+1}} \quad & U_0 = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{((C_t-C_{ht})^{\mu_c}(H_t-H_{ht})^{\mu_h})^{1-\eta}-1}{1-\eta}, \\
\text{s.t.} \quad & Y_t = a_y^{(1-\alpha_y)t}Z_{yt}K_{yt}^{\alpha_y}, \\
& Y_t = C_t + I_t, \\
& D_t = a_d^{(1-\alpha_d)t}Z_{dt}K_{dt}^{\alpha_d}, \\
& I_t = I_{dt} + I_{yt}, \\
& K_{yt+1} = (1-\delta_k)K_{yt} + \Phi_y\left(\frac{I_{yt}}{K_{yt}}\right)K_{yt}, \\
& K_{dt+1} = (1-\delta_k)K_{dt} + \Phi_d\left(\frac{I_{dt}}{K_{dt}}\right)K_{dt}, \\
& H_{t+1} = (1-\delta_h)H_t + D_t^{1-\phi}.
\end{aligned} \tag{3.C.3}$$

where $\eta, \alpha_y, \alpha_d, a_y, a_d, \mu_c, \mu_h > 0, \mu_c + \mu_h = 1, \delta_k \in [0,1], \delta_h \in [0,1]$ and $\Phi_y(x) = \frac{\varphi_{y,1}}{1-\kappa_y}x^{1-\kappa_y} + \varphi_{y,2}$ and Φ_d analogously.

First, if labor supply is exogenous, the system of equations derived from the optimization problem (3.C.3) for an equilibrium in period t reads

$$\Lambda_t = \mu_c(C_t - C_{ht})^{\mu_c(1-\eta)-1}(H_t - H_{ht})^{\mu_h(1-\eta)}, \tag{3.C.4a}$$

$$Y_t = a_y^{(1-\alpha_y)t}Z_{y,t}K_{y,t}^{\alpha_y}, \tag{3.C.4b}$$

$$D_t = a_d^{(1-\alpha_d)t}Z_{d,t}K_{d,t}^{\alpha_d}, \tag{3.C.4c}$$

$$Y_t = C_t + I_{y,t} + I_{d,t}, \tag{3.C.4d}$$

$$r_{y,t} = \alpha_y\frac{Y_t}{K_{y,t}}, \tag{3.C.4e}$$

$$r_{d,t} = \alpha_d P_{d,t}\frac{D_t}{K_{d,t}}, \tag{3.C.4f}$$

$$P_{h,t} = \frac{P_{d,t}}{1-\phi} D_t^{\phi}, \tag{3.C.4g}$$
$$q_{y,t} = \frac{1}{\Phi_y'\left(\frac{I_{y,t}}{K_{y,t}}\right)}, \tag{3.C.4h}$$
$$q_{d,t} = \frac{1}{\Phi_d'\left(\frac{I_{d,t}}{K_{d,t}}\right)}, \tag{3.C.4i}$$
$$K_{y,t+1} = (1-\delta_k)K_{y,t} + \Phi_y\left(\frac{I_{y,t}}{K_{y,t}}\right)K_{y,t}, \tag{3.C.4j}$$
$$K_{d,t+1} = (1-\delta_k)K_{d,t} + \Phi_d\left(\frac{I_{d,t}}{K_{d,t}}\right)K_{d,t}, \tag{3.C.4k}$$
$$H_{t+1} = (1-\delta_h)H_t + D_t^{1-\phi}, \tag{3.C.4l}$$
$$C_{h,t+1} = \chi_c C_t, \tag{3.C.4m}$$
$$H_{h,t+1} = \chi_h H_t, \tag{3.C.4n}$$
$$q_{y,t} = \mathbb{E}_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left(r_{y,t+1} - \frac{I_{y,t+1}}{K_{y,t+1}} + q_{y,t+1}\left(1-\delta_k + \Phi_y\left(\frac{I_{y,t+1}}{K_{y,t+1}}\right)\right)\right)\right], \tag{3.C.4o}$$
$$q_{d,t} = \mathbb{E}_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left(r_{d,t+1} - \frac{I_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}\left(1-\delta_k + \Phi_d\left(\frac{I_{d,t+1}}{K_{d,t+1}}\right)\right)\right)\right], \tag{3.C.4p}$$
$$P_{h,t} = \mathbb{E}_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\mu_h}{\mu_c}\frac{C_{t+1}-C_{h,t+1}}{H_{t+1}-H_{h,t+1}} + P_{h,t+1}(1-\delta_h)\right)\right], \tag{3.C.4q}$$

given the state variables $K_{y,t}, K_{d,t}, H_t, C_{h,t}, H_{h,t}, Z_{y,t}$ and $Z_{d,t}$. Additionally, the log of productivity follows the exogenous AR(1)-process

$$\ln Z_{y,t+1} = \rho_y \ln Z_{y,t} + \epsilon_{y,t+1},$$
$$\ln Z_{d,t+1} = \rho_d \ln Z_{d,t} + \epsilon_{d,t+1},$$
$$\begin{pmatrix}\epsilon_{y,t+1}\\ \epsilon_{d,t+1}\end{pmatrix} \sim \text{iidN}\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\sigma_y^2 & 0\\ 0 & \sigma_d^2\end{pmatrix}\right).$$

Finally, GDP is defined by

$$\text{GDP}_t = Y_t + P_{d,t}D_t + +MRS_t^{H,C}H_t, \quad \text{where } MRS_t^{H,C} = \frac{\mu_h}{\mu_c}\frac{C_t - C_{ht}}{H_t - H_{ht}}.$$

We re-scale the variables by $k_{x,t} := \frac{K_{x,t}}{a_y^t}$, $h_t := \frac{H_t}{a_y^{(1-\phi)\alpha_d t} a_d^{(1-\phi)(1-\alpha_d)t}}$, $c_{h,t} := \frac{C_{h,t}}{a_y^t}$, $h_{h,t} := \frac{H_{h,t}}{a_y^{(1-\phi)\alpha_d t} a_d^{(1-\phi)(1-\alpha_d)t}}$, $y_t := \frac{Y_t}{a_y^t}$, $c_t := \frac{C_t}{a_y^t}$, $i_{x,t} := \frac{I_{x,t}}{a_y^t}$, $d_t := \frac{D_t}{a_y^{\alpha_d t} a_d^{(1-\alpha_d)t}}$, $p_{h,t} := \frac{a_d^{(1-\phi)(1-\alpha_d)t}}{a_y^{(1-(1-\phi)\alpha_d)t}} P_{h,t}$, $p_{d,t} := \frac{a_d^{(1-\alpha_d)t}}{a_y^{(1-\alpha_d)t}} P_{d,t}$, and $\lambda_t := \frac{\Lambda_t}{a_y^{((\mu_c+(1-\phi)\alpha_d\mu_h)(1-\eta)-1)t} a_d^{(1-\phi)(1-\alpha_d)\mu_h(1-\eta)t}}$. Hence, system (3.C.4) in scaled variables reads

$$\lambda_t = \mu_c(c_t - c_{ht})^{\mu_c(1-\eta)-1}(h_t - h_{ht})^{\mu_h(1-\eta)}, \tag{3.C.5a}$$
$$y_t = Z_{y,t}k_{y,t}^{\alpha_y}, \tag{3.C.5b}$$
$$d_t = Z_{d,t}k_{d,t}^{\alpha_d}, \tag{3.C.5c}$$
$$y_t = c_t + i_{y,t} + i_{d,t}, \tag{3.C.5d}$$
$$r_{y,t} = \alpha_y\frac{y_t}{k_{y,t}}, \tag{3.C.5e}$$

$$r_{d,t} = \alpha_d p_{d,t} \frac{d_t}{k_{d,t}}, \tag{3.C.5f}$$

$$p_{h,t} = \frac{p_{d,t}}{1-\phi} d_t^{\phi}, \tag{3.C.5g}$$

$$q_{y,t} = \frac{1}{\Phi_y'\left(\frac{i_{y,t}}{k_{y,t}}\right)}, \tag{3.C.5h}$$

$$q_{d,t} = \frac{1}{\Phi_d'\left(\frac{i_{d,t}}{k_{d,t}}\right)}, \tag{3.C.5i}$$

$$a_y k_{y,t+1} = (1-\delta_k)k_{y,t} + \Phi_y\left(\frac{i_{y,t}}{k_{y,t}}\right)k_{y,t}, \tag{3.C.5j}$$

$$a_y k_{d,t+1} = (1-\delta_k)k_{d,t} + \Phi_d\left(\frac{i_{d,t}}{k_{d,t}}\right)k_{d,t}, \tag{3.C.5k}$$

$$\left(a_y^{\alpha_d} a_d^{(1-\alpha_d)}\right)^{1-\phi} h_{t+1} = (1-\delta_h)h_t + d_t^{1-\phi}, \tag{3.C.5l}$$

$$a_y c_{h,t+1} = \chi_c c_t, \tag{3.C.5m}$$

$$\left(a_y^{\alpha_d} a_d^{(1-\alpha_d)}\right)^{1-\phi} h_{h,t+1} = \chi_h h_t, \tag{3.C.5n}$$

$$q_{y,t} = \mathbb{E}_t\left[\beta a_y^{(\mu_c+(1-\phi)\alpha_d\mu_h)(1-\eta)-1} a_d^{(1-\phi)(1-\alpha_d)\mu_h(1-\eta)} \frac{\lambda_{t+1}}{\lambda_t}\right. \tag{3.C.5o}$$

$$\left.\left(r_{y,t+1} - \frac{i_{y,t+1}}{k_{y,t+1}} + q_{y,t+1}\left(1-\delta_k+\Phi_y\left(\frac{i_{y,t+1}}{k_{y,t+1}}\right)\right)\right)\right], \tag{3.C.5p}$$

$$q_{d,t} = \mathbb{E}_t\left[\beta a_y^{(\mu_c+(1-\phi)\alpha_d\mu_h)(1-\eta)-1} a_d^{(1-\phi)(1-\alpha_d)\mu_h(1-\eta)} \frac{\lambda_{t+1}}{\lambda_t}\right. \tag{3.C.5q}$$

$$\left.\left(r_{d,t+1} - \frac{i_{d,t+1}}{k_{d,t+1}} + q_{d,t+1}\left(1-\delta_k+\Phi_d\left(\frac{i_{d,t+1}}{k_{d,t+1}}\right)\right)\right)\right], \tag{3.C.5r}$$

$$p_{h,t} = \mathbb{E}_t\left[\beta a_y^{(\mu_c+(1-\phi)\alpha_d\mu_h)(1-\eta)-(1-\phi)\alpha_d} a_d^{(1-\phi)(1-\alpha_d)(\mu_h(1-\eta)-1)} \frac{\lambda_{t+1}}{\lambda_t}\right. \tag{3.C.5s}$$

$$\left.\left(\frac{\mu_h}{\mu_c}\frac{c_{t+1}-c_{h,t+1}}{h_{t+1}-h_{h,t+1}} + p_{h,t+1}(1-\delta_h)\right)\right], \tag{3.C.5t}$$

Endogenous labor supply:

$$\begin{aligned}
\max_{\dots,N_{yt},N_{dt}} U_0 &= \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{((C_t-C_{ht})^{\mu_c}(H_t-H_{ht})^{\mu_h}((1-N_t)-(1-N_{ht}))^{\mu_n})^{1-\eta}-1}{1-\eta},\\
\text{s.t.}\quad & Y_t = Z_{yt}K_{yt}^{\alpha_y}(a_y^t N_{yt})^{1-\alpha_y},\\
& Y_t = C_t + I_t,\\
& D_t = Z_{dt}K_{dt}^{\alpha_d}(a_d^t N_{dt})^{1-\alpha_d},\\
& N_t = N_{dt} + N_{yt},\\
& N_t \leq 1\\
& \dots
\end{aligned} \tag{3.C.6}$$

where now $\mu_c, \mu_h, \mu_n > 0, \mu_c + \mu_h + \mu_n = 1$.

If labor supply is endogenous in the model as in (3.C.6), the system of equations defining an equilibrium in the scaled variables remains as in (3.C.5) with the following adjustments to the production functions and to the marginal utility of consumption and with the additional equations pinning down labor supply

$$y_t = Z_{y,t} k_{y,t}^{\alpha_y} N_{y,t}^{1-\alpha_y}, \tag{3.C.7a}$$
$$d_t = Z_{d,t} k_{d,t}^{\alpha_d} N_{d,t}^{1-\alpha_d}, \tag{3.C.7b}$$
$$\lambda_t = \mu_c (c_t - c_{ht})^{\mu_c(1-\eta)-1} (h_t - h_{ht})^{\mu_h(1-\eta)} ((1-N_t) - (1-N_{h,t}))^{\mu_n(1-\eta)}, \tag{3.C.7c}$$
$$(1-\alpha_y)\frac{y_t}{N_{y,t}} = \frac{\mu_n}{\mu_c} \frac{c_t - c_{h,t}}{(1-N_t) - (1-N_{h,t})}, \tag{3.C.7d}$$
$$(1-\alpha_d) p_{d,t} \frac{d_t}{N_{d,t}} = \frac{\mu_n}{\mu_c} \frac{c_t - c_{h,t}}{(1-N_t) - (1-N_{h,t})}, \tag{3.C.7e}$$
$$N_t = N_{y,t} - N_{d,t}, \tag{3.C.7f}$$
$$N_{h,t+1} = 1 - \chi_n (1 - N_t). \tag{3.C.7g}$$

Limited labor mobility Finally, if the household is unable to adapt his labor supply in response to technology shocks but is committed to working hours that are contracted sector-specifically one period ahead, the conditions in (3.C.7d) and (3.C.7e) must be adjusted to

$$\mathbb{E}_t \left[\lambda_{t+1} \left((1-\alpha_y) \frac{y_{t+1}}{N_{y,t+1}} - \frac{\mu_n}{\mu_c} \frac{c_{t+1} - c_{h,t+1}}{(1-N_{t+1}) - (1-N_{h,t+1})} \right) \right] = 0, \tag{3.C.8a}$$
$$\mathbb{E}_t \left[\lambda_{t+1} \left((1-\alpha_d) p_{d,t+1} \frac{d_{t+1}}{N_{d,t+1}} - \frac{\mu_n}{\mu_c} \frac{c_{t+1} - c_{h,t+1}}{(1-N_{t+1}) - (1-N_{h,t+1})} \right) \right] = 0. \tag{3.C.8b}$$

3.D Housing with disaster risk

We present the details for the model with disaster risk from section 3.4.

Disaster Risk The economy faces a risk for great disasters which are introduced through an exogenous shock in form of a binary variable b_t which indicates disasters in case of $b_t = 1$ while $b_t = 0$ in normal times. Disasters reduce productivity but also partly destroy the stock of productive capital and of residential structures (see below). Following Gourio (2012) disasters appear with time-varying probability and size. More specifically, we assume that

$$P(b_{t+1} = 1 | b_t = 0) = \min\{p_t, 1\}, \; P(b_{t+1} = 0 | b_t = 0) = 1 - \min\{p_t, 1\}$$

where the log of p_t follows an AR(1)-process

$$\ln p_{t+1} = (1-\rho_p) \ln \bar{p} + \rho_p \ln p_t + \epsilon_{p,t+1}, \; \epsilon_{p,t} \sim \text{iidN}(0, \sigma_p^2).$$

Additionally, disasters remain persistent with probability no less than $q \in (0,1)$ so that

$$P(b_{t+1} = 1 | b_t = 1) = \max\{q, \min\{p_t, 1\}\}, \; P(b_{t+1} = 0 | b_t = 1) = 1 - \max\{q, \min\{p_t, 1\}\}.$$

Finally, the disaster size $1 - e^{\omega_{t+1}}$ at which productivity, productive capital and residential structures are destroyed by a disaster also evolves stochastically according to

$$\omega_t := \bar{\omega} e^{\hat{\omega}_t},$$
$$\hat{\omega}_{t+1} = \rho_\omega \hat{\omega}_t + \epsilon_{\omega,t+1}, \; \epsilon_{\omega,t} \sim \text{iidN}\left(0, \sigma_\omega^2\right),$$

where $\bar{\omega} < 0$. We slightly deviate from the treatment in Gourio (2012) in the specification of the process governing the disaster size and allow autocorrelation but restrict outcomes to $\omega_t < 0$ so that disasters always have negative effects. The specification is similar to Fernández-Villaverde and Levintal (2018).

Representative Household The household derives utility from a composite good $\tilde{C}_t$ that is represented by a Cobb-Douglas aggregate consisting of consumption C_t, housing H_t and leisure $1-N_t$, i.e.

$$\tilde{C}_t := C_t^{\mu_c} H_t^{\mu_h} (1-N_t)^{1-\mu_c-\mu_h}.$$

We assume that the household's preferences over streams of the composite good are described by a recursive utility function, as introduced by Epstein and Zin (1989), of the form

$$\tilde{V}_t = \left[(1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t \tilde{V}_{t+1}^{1-\gamma})^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}},$$

where ψ is the household's EIS and γ the coefficient of RRA. Note however that γ and ψ describe the household's RRA and EIS with respect to the composite good $\tilde{C}$. Since the composite good aggregator is of the Cobb-Douglas type, the consumption-based RRA is given by $\mu_c\gamma$ and the consumption-based EIS reads $\frac{1}{1-\mu_c(1-1/\psi)}$.[19] For easier notation we define $V_t := \tilde{V}_t^{1-1/\psi}$ which satisfies the recursion

$$V_t = (1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}},$$

where we use, similar to Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012), the notation

$$\theta := 1 - \frac{1-\gamma}{1-\frac{1}{\psi}}.$$

In the case where $\theta = 0$, the RRA equals the reciprocal of the EIS and the household's utility reduces to the 'classical' expected discounted sum of within period CRRA utilities. Hence, θ can also be interpreted as the deviation from this 'classic' case. The representative household supplies labor services N_t and capital services K_t and receives wages W_t and capital rents r_t. He buys consumption goods C_t and invests in productive capital I_t and new houses $H_{\text{new},t}$ with relative price $P_{h,t}$. Hence, his budget constraint reads

$$W_t N_t + r_t K_t = C_t + I_t + P_{h,t} H_{\text{new},t}.$$

We assume capital adjustment costs as in Jermann (1998). Moreover, disasters result in the destruction of a fraction $1-e^{\omega_{t+1}}$ of the stocks of capital and residential structures so that the stocks accumulate according to

$$K_{t+1} = e^{\omega_{t+1} b_{t+1}} \underbrace{\left((1-\delta_k)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t\right)}_{=:K^\star_{t+1}},$$

$$H_{t+1} = e^{\omega_{t+1} b_{t+1}(1-\phi)} \underbrace{\left((1-\delta_h)H_t + D_t^{1-\phi}\right)}_{=:H^\star_{t+1}},$$

where $\delta_k, \delta_h \in [0,1]$ and $\Phi(x) := \frac{\varphi_1}{1-\kappa}x^{1-\kappa} + \varphi_2$. The household maximizes life-time utility V_t subject to his budget constraint and subject to the laws of accumulation for capital and housing.

[19]See Swanson (2012) and Heiberger and Ruf (2019).

Hence, the first order conditions for the household are given by

$$
\begin{aligned}
W_t &= \frac{\mu_n}{\mu_c}\frac{C_t}{1-N_t},\\
q_t &= \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)},\\
q_t &= \mathbb{E}_t\left[M_{t+1}e^{\omega_{t+1}b_{t+1}}\left(r_{t+1}+q_{t+1}\left(1-\delta_k+\Phi\left(\frac{I_{t+1}}{K_{t+1}}\right)-\Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right)\right)\right],\\
P_{h,t} &= \mathbb{E}_t\left[M_{t+1}e^{(1-\phi)\omega_{t+1}b_{t+1}}\left(\frac{\mu_h}{\mu_c}\frac{C_{t+1}}{H_{t+1}}+P_{h,t+1}(1-\delta_h)\right)\right],
\end{aligned}
$$

where M_{t+1} denotes the model's stochastic discount factor

$$
M_{t+1} := \beta\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left(\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\theta})^{1/(1-\theta)}}\right)^{-\theta} \quad \text{with} \quad \Lambda_t := \mu_c\frac{\tilde{C}_t^{1-\frac{1}{\psi}}}{C_t}.
$$

Representative Firm The firm produces output Y_t from labor N_t and capital services K_t according to the production function

$$
Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha}.
$$

Labor augmenting technical progress A_t grows stochastically and is damaged during disasters such way that

$$
\begin{aligned}
A_{t+1} &= A_t a e^{z_{t+1}+\omega_{t+1}b_{t+1}},\\
z_{t+1} &= \rho_z z_t + \epsilon_{z,t+1},\ \ \epsilon_{z,t} \sim \text{iidN}(0,\sigma_z^2).
\end{aligned}
$$

The firm's first order conditions from maximization of profits $Y_t - W_t N_t - r_t K_t$ subject to the production function read

$$
\begin{aligned}
W_t &= (1-\alpha)\frac{Y_t}{N_t},\\
r_t &= \alpha\frac{Y_t}{K_t}.
\end{aligned}
$$

Construction Sector Finally, residential investments D_t are combined with a fixed factor land $L_t \equiv 1$ in order to form new houses according to

$$
H_{\text{new},t} = D_t^{1-\phi}L_t^{\phi}.
$$

Maximization of profits $P_{h,t}H_{\text{new},t} - D_t - P_{l,t}L_t$ yields the first order conditions

$$
\begin{aligned}
P_{h,t} &= \frac{1}{1-\phi}D_t^{\phi},\\
P_{l,t} &= \phi P_{h,t} D_t^{1-\phi}.
\end{aligned}
$$

General Equilibrium Summing up, in any period t the economy's equilibrium is characterized by the following system of equations

$$
A_t = A_{t-1}e^{z_t+\omega_t b_t}, \tag{3.D.1a}
$$

$$K_t = e^{\omega_t b_t} K_t^\star, \tag{3.D.1b}$$
$$H_t = e^{(1-\phi)\omega_t b_t} H_t^\star, \tag{3.D.1c}$$
$$Y_t = K_t^\alpha (A_t N_t)^{1-\alpha}, \tag{3.D.1d}$$
$$r_t = \alpha \frac{Y_t}{K_t}, \tag{3.D.1e}$$
$$W_t = (1-\alpha)\frac{Y_t}{N_t}, \tag{3.D.1f}$$
$$W_t = \frac{\mu_n}{\mu_c}\frac{C_t}{1-N_t}, \tag{3.D.1g}$$
$$Y_t = C_t + I_t + D_t, \tag{3.D.1h}$$
$$\Lambda_t = \mu_c C_t^{\mu_c(1-\frac{1}{\psi})-1}(1-N_t)^{\mu_n(1-\frac{1}{\psi})} H_t^{\mu_h(1-\frac{1}{\psi})}, \tag{3.D.1i}$$
$$P_{h,t} = \frac{1}{1-\phi} D_t^\phi, \tag{3.D.1j}$$
$$q_t = \frac{1}{\varphi_1}\left(\frac{I_t}{K_t}\right)^\kappa, \tag{3.D.1k}$$
$$K_{t+1}^\star = (1-\delta_k)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t, \tag{3.D.1l}$$
$$H_{t+1}^\star = (1-\delta_h)H_t + D_t^{1-\phi}, \tag{3.D.1m}$$
$$q_t = \mathbb{E}_t\left[M_{t+1}e^{\omega_{t+1}b_{t+1}}\left(r_{t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\left(1-\delta_k + \Phi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right)\right], \tag{3.D.1n}$$
$$P_{h,t} = \mathbb{E}_t\left[M_{t+1}e^{(1-\phi)\omega_{t+1}b_{t+1}}\left(\frac{\mu_h}{\mu_c}\frac{C_{t+1}}{H_{t+1}} + P_{h,t+1}(1-\delta_h)\right)\right], \tag{3.D.1o}$$
$$V_t = (1-\beta)(C_t^{\mu_c}(1-N_t)^{\mu_n}H_t^{\mu_h})^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}, \tag{3.D.1p}$$

given the state variables $K_t^\star, H_t^\star, z_t, \omega_t, p_t$ and b_t. The stochastic discount factor satisfies

$$M_{t+1} := \beta\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\left(\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\theta})^{1/(1-\theta)}}\right)^{-\theta}$$

Moreover, the exogenous state variables are governed by the stochastic processes

$$z_{t+1} = \rho_z z_t + \epsilon_{z,t+1},\ \epsilon_{z,t} \sim \text{iidN}(0,\sigma_z^2), \tag{3.D.2a}$$
$$\ln p_{t+1} = (1-\rho_p)\ln \bar{p} + \rho_p \ln p_t + \epsilon_{p,t+1},\ \epsilon_{p,t} \sim \text{iidN}(0,\sigma_p^2), \tag{3.D.2b}$$
$$\omega_t := \bar{\omega}e^{\hat{\omega}_t},\ \hat{\omega}_{t+1} = \rho_\omega \hat{\omega}_t + \epsilon_{\omega,t+1},\ \epsilon_{\omega,t} \sim \text{iidN}\left(0,\sigma_\omega^2\right), \tag{3.D.2c}$$
$$P(b_{t+1}=1|b_t=0) = \min\{p_t,1\},\ P(b_{t+1}=1|b_t=1) = \max\{q,\min\{p_t,1\}\}. \tag{3.D.2d}$$

Finally, we define GDP as the sum of consumption, both investment types and the implicit rent from housing

$$\text{GDP}_t = Y_t + MRS_t^{H,C}H_t, \ \text{ where } MRS_t^{H,C} := \frac{\mu_h}{\mu_c}\frac{C_t}{H_t}.$$

We scale the variables in terms of technology A_{t-1} by $a_t := \frac{A_t}{A_{t-1}}$, $k_t^\star := \frac{K_t^\star}{A_{t-1}}$, $h_t^\star := \frac{H_t^\star}{A_{t-1}^{1-\phi}}$, $k_t := \frac{K_t}{A_{t-1}}$, $h_t := \frac{H_t}{A_{t-1}^{1-\phi}}$, $y_t := \frac{Y_t}{A_{t-1}}$, $c_t := \frac{C_t}{A_{t-1}}$, $i_t := \frac{I_t}{A_{t-1}}$, $d_t := \frac{D_t}{A_{t-1}}$, $w_t := \frac{W_t}{A_{t-1}}$, $p_{h,t} := \frac{P_{h,t}}{A_{t-1}^\phi}$, $\lambda_t := \frac{\Lambda_t}{A_{t-1}^{(\mu_c+(1-\phi)\mu_h)(1-1/\psi)-1}}$, and $v_t := \frac{V_t}{A_{t-1}^{(\mu_c+(1-\phi)\mu_h)(1-1/\psi)}}$. Hence, the system of equations (3.D.1) can be written equivalently in terms of the scaled variables as

$$a_t = e^{z_t+\omega_t b_t}, \tag{3.D.3a}$$
$$k_t = e^{\omega_t b_t} k_t^\star, \tag{3.D.3b}$$
$$h_t = e^{(1-\phi)\omega_t b_t} h_t^\star, \tag{3.D.3c}$$
$$y_t = k_t^\alpha (a_t N_t)^{1-\alpha}, \tag{3.D.3d}$$
$$r_t = \alpha \frac{y_t}{k_t}, \tag{3.D.3e}$$
$$w_t = (1-\alpha)\frac{y_t}{N_t}, \tag{3.D.3f}$$
$$w_t = \frac{\mu_n}{\mu_c}\frac{c_t}{1-N_t}, \tag{3.D.3g}$$
$$y_t = c_t + i_t + d_t, \tag{3.D.3h}$$
$$\lambda_t = \mu_c c_t^{\mu_c(1-\frac{1}{\psi})-1}(1-N_t)^{\mu_n(1-\frac{1}{\psi})} h_t^{\mu_h(1-\frac{1}{\psi})}, \tag{3.D.3i}$$
$$p_{h,t} = \frac{1}{1-\phi} d_t^\phi, \tag{3.D.3j}$$
$$q_t = \frac{1}{\varphi_1}\left(\frac{i_t}{k_t}\right)^\kappa, \tag{3.D.3k}$$
$$a_t k_{t+1}^\star = (1-\delta_k)k_t + \Phi\left(\frac{i_t}{k_t}\right)k_t, \tag{3.D.3l}$$
$$a_t^{1-\phi} h_{t+1}^\star = (1-\delta_h)h_t + d_t^{1-\phi} \tag{3.D.3m}$$
$$q_t = \mathbb{E}_t\left[a_t^{(\mu_c+(1-\phi)\mu_h)(1-1/\psi)-1} m_{t+1} e^{\omega_{t+1}b_{t+1}}\left(r_{t+1} - \frac{i_{t+1}}{k_{t+1}} + q_{t+1}\left(1-\delta_k+\Phi\left(\frac{i_t}{k_t}\right)\right)\right)\right], \tag{3.D.3n}$$
$$p_{h,t} = \mathbb{E}_t\left[a_t^{(\mu_c+(1-\phi)\mu_h)(1-1/\psi)-1+\phi} m_{t+1} e^{(1-\phi)\omega_{t+1}b_{t+1}}\left(\frac{\mu_h}{\mu_c}\frac{c_{t+1}}{h_{t+1}} + p_{h,t+1}(1-\delta_h)\right)\right], \tag{3.D.3o}$$
$$v_t = (1-\beta)(c_t^{\mu_c}(1-N_t)^{\mu_n} h_t^{\mu_h})^{1-\frac{1}{\psi}} + a_t^{(\mu_c+(1-\phi)\mu_h)(1-1/\psi)}\beta(\mathbb{E}_t v_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}, \tag{3.D.3p}$$

where

$$m_{t+1} := \beta\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{v_{t+1}}{(\mathbb{E}_t v_{t+1}^{1-\theta})^{1/(1-\theta)}}\right)^{-\theta}.$$

Solution Method First, note that given period t's scaled state variables $k_t^\star, h_t^\star, z_t, \omega_t, p_t$ and b_t and the control variables for labor supply N_t, house prices $p_{h,t}$ and the value function v_t, all other period t variables as well as next period's endogenous state variables can be easily computed from equations (3.D.3a)-(3.D.3m). We approximate the policy functions for N_t, $p_{h,t}$ and the value function v_t by linear combinations of Chebyshev polynomials. We compute the coefficients in the linear combinations such way that the Euler equations (3.D.3n) and (3.D.3o) and the recursive equation (3.D.3p) for the value function are satisfied exactly at a sparse grid of collocation points (see Judd, Maliar, Maliar, and Valero (2014) and Heer and Maussner (2009) for details). Thereby, the expectations with respect to normally distributed random variables are computed by Gauss-Hermite quadrature.

Chapter 4

Business cycle accounting for the German fiscal stimulus program during the Great Recession

— Daniel Fehrle and Johannes Huber—

4.1 Introduction

In response to the Great Recession in 2008 and 2009, the German government, like many others, launched an expansive fiscal stimulus program. This policy intervened on different markets by increasing transfers and government spending, decreasing tax rates and social contributions and expanding short-time work possibilities. Particularly noteworthy is the German cash for clunkers program, since this car subsidy affected one of Germany's core industries and was internationally incomparably large (5 Billion € or 0.2 percent of Gross Domestic Product (GDP)). Altogether, the program amounted to 82 billion € or 3.2 percent of GDP. These considerable expenditures raise the following questions: What are the consequences of these measures for macroeconomic markets and how effective was this program for aggregated output?

Such questions are difficult to answer, which is why fiscal stimuli might be the most controversially discussed anti-cyclical measures. To address them, there are basically two approaches (see e.g. Hebous (2010)): The first is to model a theoretical framework with deep structural equations, parameters, and shocks. An arbitrary number of shocks describes changes in fiscal policy, and impulse response functions as well as multipliers illustrate the consequences. Since the structure, the parametrization, and, at least partly, the parameter values ground on assumptions, the results are assumption-driven. The second approach bases on statistical models, in particular vector autoregressions (VARs). They are less theoretical and, in comparison to many of the former models, can be estimated with classical techniques. Unfortunately, in general it is impossible to distinguish between market distortions and the agent's responses to these distortions. This makes it rather impractical to study the effects of the various market interventions. Instead of selecting from these two approaches, we apply a third option, which we describe as kind of a middle course. By employing the Business Cycle Accounting (BCA) approach as proposed by Chari, Kehoe, and McGrattan (2007) and revisited by Brinca, Chari, Kehoe, and McGrattan (2016), we investigate the impact of the Great Recession during 2008 and 2009 in Germany, its aftermath, the impact of monetary policy, and in particular, the effects of the German stimulus program.

The BCA framework is based on the benchmark Real Business Cycle (RBC) model, which is extended by time-varying distortions in nearly every market, the so-called "prototype economy". Chari, Kehoe, and McGrattan (2007) interpret the origins of these market distortions as taxes, nominal and real frictions, changes in expectations, etc. and call them "wedges". In contrast to most medium or large scale Dynamic Stochastic General Equilibrium (DSGE) models, the mechanisms underlying these distortions are not structural. They are parameterized like taxes, technology, or government spending and are driven by a reduced-form Markov process.[1] Commonly this process is specified by a VAR(1). Using time series data one can estimate the parameters of the VAR process and measure the values of the wedges. These measured wedges are fed back into the model one by one, to assess the contribution of each wedge to the business cycle. In a nutshell, BCA is the fully developed "...through the lens of a neoclassical model"-approach.[2] The slim theoretical framework and the applicability of classical estimation techniques, in this instance maximum-likelihood estimation (MLE), minimizes the number of assumptions required and thus the results are less assumption-driven. Nevertheless, one can distinguish between market distortions and the agents responses.

To increase the practicability of BCA in general and make it more suitable for the study of the German stimulus program in particular, we differ from Chari, Kehoe, and McGrattan (2007) in our "prototype-economy", in our estimation methodology, and in our mapping strategy.

Prototype-economy: We extend the benchmark model for the following reasons in three ways. First, the wedges include a long- and a short-run component. This allows to differentiate between growth and business cycle accounting. Since the German reunification, subaggregates of demand grew at different rates. Without growth accounting, the underlying stochastic process is non-stationary. Chari, Kehoe, and McGrattan (2007) set a common growth rate unfoundedly for all countries equal to 1.6 percent. Brinca, Chari, Kehoe, and McGrattan (2016) detrend in such a way that the average trend-adjusted log output of the economy under consideration is equal zero. The latter makes the estimation procedure more robust. Our approach can be seen as a further stage.[3] Second, we distinguish between government spending and net exports. This enables a government spending analysis and accounts for the fact that German industry is strongly depended on foreign trade. Third, we exclude durable consumption goods from aggregated investment in order to consider the cash for clunkers program separately. After all, the model includes the following wedges: *government consumption, durables, investment, labor, net exports, and efficiency*. Previous work already extends the benchmark model in various ways, e.g. Šustek (2011) includes an asset market and a monetary policy wedge.

Estimation: We estimate two structural parameters and all parameters of the VAR process using MLE, in sum 59, and identify the wedges with Kalman-smoothing. MLE in this context is difficult, e.g. Gerth and Otsu (2018) report unsolved problems concerning likelihood optimization and BCA.[4] As many others, they avoid the problem by switching to Bayesian estimation. As we argue, Bayesian methods are impracticable for BCA, because the reduced-form process is highly abstract and thus, it seems impossible to make any a-priori assumptions. Furthermore, Brinca, Iskrev, and Loria (2018) argue that weak identification associated with parameters of the VAR process is negligible in the context of BCA. Unfortunately, this does not hold for structural parameters. We introduce a reliable and quick procedure to locate the maximum of the likelihood function. Using this procedure, it is a feasible exercise to apply tools that help overcome problems of weak identification, namely plotting the likelihood contour, detecting the global maximum, and executing robustness checks, all with respect to the uncertain structural parameters.

The procedure can be summarized as follows: In advance, we make sure that all uncertain parameters are locally strictly identified according to the strategy of Iskrev (2010). Then, we

[1] Note that Chari, Kehoe, and McGrattan (2009) argue that also some of the shocks in medium or large scale DSGE models, i.e. New Keynesian models, are rather reduced-form than structural.

[2] This long-lasting approach was established by Solow (1957). To name but a few more recent applications: Kehoe and Prescott (2002), Ohanian (2010), Lu (2012), Cho and Doblas-Madrid (2013), Karabarbounis (2014) or Hansen and Ohanian (2016).

[3] Note that growth accounting is implicitly applied whenever different time series are detrended by univariate filters, such as the HP-filter, the Hamilton filter or the Baxton-King filter. DeJong and Dave (2011, Chapter 6.1) suggest a general procedure to estimate a common linear trend. Even by applying this strategy, the estimated process lacks stationarity here.

[4] Gerth and Otsu (2018) do not account for growth, which potentially explains the problem.

maximize the likelihood function, which we receive from a Kalman recursion, assuming that the initial states are fixed and known in their long-run equilibrium. As Huber (2020) shows, this initialization is in line with Chari, Kehoe, and McGrattan (2007) and provides two advantages, i) the computation of the likelihood function can be vectorized and ii) an analytical and unique solution exists for the maximizing conditional covariance matrix. Further, Huber (2020) proves that the average of this likelihood function converges pointwise towards the average of a likelihood function received from a Kalman recursion initialized with the unconditional first and second moments. Thus, we use the first parameter estimation only as a guess for the actual estimation based on the more common, unconditional likelihood function. As mentioned, we complete the process by determining the wedges with Kalman-smoothing.

Mapping: Chari, Kehoe, and McGrattan (2007) map different types of structural frictions towards the reduced-form wedges, which they call "equivalent results". We map the particular measures of the fiscal stimulus program and monetary policy in a similar manner and analyze whether these interventions can explain counter-cyclical behavior. This follows Mulligan (2005) who initiates the study of policy interventions as reduced-form errors of RBC models, and Kersting (2008) who initiates the mapping of political measures, namely the 1980's U.K. labor market reforms, towards the wedges inside the BCA framework.

Our findings suggest that the crisis was mainly driven by the efficiency wedge, followed by the net exports and the investment wedge. The government consumption wedge and especially the durables wedge acted counter-cyclically. Furthermore, the labor wedge induced a fast recovery. The results are robust except for the investment wedge.

We attribute the counter-cyclicality of the durables wedge to the cash for clunkers program, which is equivalent to a durable good subsidy. Since the expenditures for government consumption were higher than for the cash for clunkers program and the effects were similar, subsidies for durable goods stimulated aggregated demand more efficiently. Mian and Sufi (2012) examine the U.S. cash for clunkers program as a representative of durables and investment subsidies using cross-section variation. They find that the program induced a large increase in car sales. Indeed, in their study, the positive effect vanishes within one year due to intertemporal substitution. In Germany, durable goods bust after the program, which suggest a similar substitution effect. However, our BCA analysis indicates that this is the transmission towards the trajectory of durables that would have occurred in the absence of the cash for clunkers program. In sum the program's effects are neither substituted entirely intra- nor intertemporally untill 2011-Q3. This is at odds with the results of a times-series analysis by Leuwer and Süssmuth (2018), who find large substitution effects. However, their work relies on the strong assumption that there were no substantial changes simultaneously to the car subsidy. Berger and Vavra (2015) investigates the households' responses to durables subsidies over the business cycle for the U.S. and find smaller effects in recessions, which is not at odds to our results, but make them more striking.

The labor market wedge induced recovery can be explained by expanded short-time work possibilities as they can decrease hiring frictions in the aftermath of recessions. Using the unemployment rate, Gehrke, Lechthaler, and Merkl (2019) argue that previous labor market reforms (so-called Hartz reforms) probably drove the labor market wedge induced recovery. Our method cannot distinguish between these explanations because both achieve equivalent results.

Similar interpretation problems concerning reduced-form shocks arise with measures of the stimulus program which we map towards the efficiency, investment, and net exports wedge. Since these wedges caused the crisis, pro-cyclical distortions exceed the effects of counter-cyclical fiscal stimulus and monetary policy measures in those markets. Hence, pro-cyclical wedges give no evidence for ineffective measures. Assuming that the fiscal stimulus program together with monetary policy were the only counter-cyclical distortions, counter-cyclical wedges give evidence for effective measures. Under this assumption, our results represent a lower bound for the impact of fiscal and monetary policy measures and the pro-cyclical distortions.

Existing BCA applications for the Great Recession in Germany by Brinca, Chari, Kehoe, and McGrattan (2016) and Gerth and Otsu (2018) suggest negligible effects of the investment wedge on the business cycle. Both treat durables and other investment goods as a composite. We get similar results, feeding back both wedges at the same time into the model. In detail, the pro-cyclicality of the investment wedge and the counter-cyclicality of the durables wedge offset each other, which is why previous work potentially underrate the importance of the investment wedge and, as a consequence, equivalent financial frictions.

Drygalla, Holtemoeller, and Kiesel (2018) as well as Gadatsch, Hauzenberger, and Stähler (2016) investigate the German fiscal stimulus program in medium-scale New-Keynesian DSGE models using Bayesian inference. They find positive but small effects on GDP and the latter finds negative effects in the aftermath of the crisis. However, neither of these studies account for durable consumption goods separately.

The remainder of the paper reads as follows. The next section sketches the German fiscal stimulus program and the monetary policy of the European Central Bank (ECB). Furthermore, we provide long-term series with focus on the crisis from 2008 till 2011 for the reunified German economy. Thereafter, we describe our version of a prototype economy. We map the single measures of the program to the wedges. In a next step, we present our calibration exercises and the estimation strategy. We show the results with a robustness and discussion section and then the paper concludes. Our Appendix presents the entire model as well as the source of our data and the corresponding manipulation.

4.2 The German case

4.2.1 The fiscal stimuli packages I and II in detail

The German fiscal stimulus program was composed of two packages. The first became effective at the end of 2008 and the second at the beginning of 2009 (Bundesgesetzblatt, 2008, 2009).

As Rosenberger (2013) describes, the first package amounted to 32 Billion € plus a loan program of 15 Billion €. The fiscal stimulus consisted of a one year's tax exemption on new cars, higher tax deductions by permitting the reducing-balance method and increasing child allowance, a lower employment insurance tax, as well as higher transfers for students and retirees.

The second stimulus package amounted to 50 Billion € plus both a loan and guarantee program of 100 Billion € and an increase of the German export credit guarantee program (Hermes cover) of about 2 Billion €. The package consisted of investments in public infrastructure, financial support for local and state authority spending, a subsidy on new cars at the amount of 2500 € per car and in total 5 Billion €, subsidies for private innovations as well as lower income taxes and social contributions. Short-time work possibilities and benefits were expanded, further training was supported, and the Federal Employment Agency increased the number of job agents.

Table 4.1 presents following calculations by the OECD (2009) for the stimulus program. The size of the fiscal stimulus program was on equal terms by reducing taxes and increasing transfers and spending. Transfers to households amounted to 0.3 percent of GDP, where the cash for clunkers composed two out of three. Extra government spending amounted to 0.8 percent of GDP. The fiscal packages amounted to 3.2 percent of GDP, excluding all measures which did not affected the national budget directly, e.g. the loan and guarantee program.

Table 4.1: Composition of the fiscal program in percent of GDP

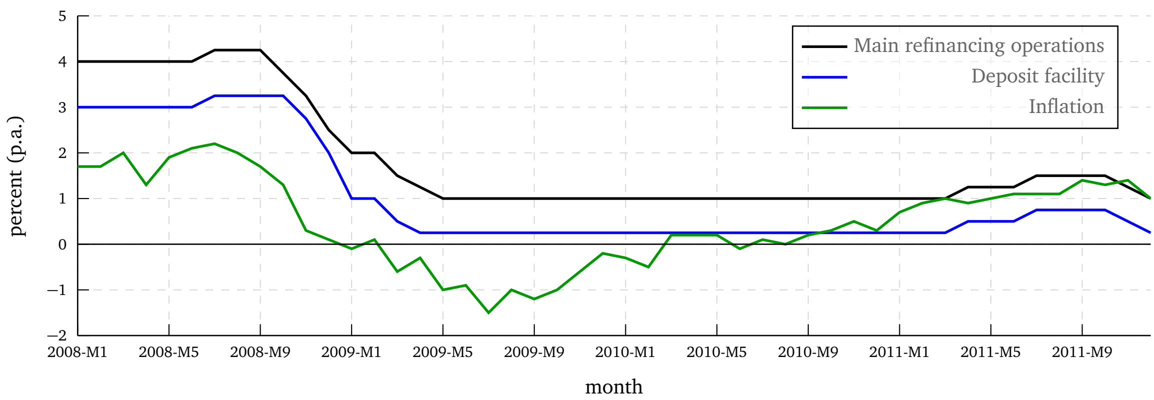
Tax	Individuals	Social Contribution	Business	Total*
	-0.6	-0.7	-0.3	-1.6
Spending	Transfers to households	Transfers to business	Government spending**	Total***
	0.3	0.3	0.8	1.6

Notes: * Including consumption tax measures. ** Final consumption + investment *** Including transfers to sub-national government. **Source:** OECD (2009).

4.2.2 Monetary policy in the Great Recession

The monetary policy of the ECB also reacted to the recession. Figure 4.1 shows the minimum bid rate on main refinancing operations and the interest rate on deposit facilities declined in the aftermath of the declined inflation rate. The former declined from 4.25 percent in mid 2008 to 1 percent by mid 2009. Both interest rates have persisted since then.

Figure 4.1: Monetary policy and usage of the deposit facility



Besides the conventional interest rate policy, the ECB applied further tools of monetary policy. Here we give a short overview of the detailed reports of the European Central Bank (2010, 2011). In October 2008 the ECB switched from a variable-rate to a fixed-rate tender, eased collateral requirements and enhanced the provision of liquidity. The ECB's Governing Council prolonged these measures several times. It decided to purchase bonds issued in the Euro area in May 2009 and launched the Security Markets Program in June 2009. This program conducted interventions on public and private debt securities markets in the Euro area. Then, in March and May 2010, the Governing Council decided to switch back and forth between a variable- and a fixed-rate tender and to intervene once again on the Euro area public and private debt securities markets. The Council determined long-term refinance operations to provide liquidity in August and October 2010.

4.2.3 Stylized facts for the German economy

Table 4.2 presents average long-run shares of subaggregates of the reunified German economy (1991–2018). Private Consumption Expenditure (PCE) account for 56 percent, whereby durables account for 6 percent and non-durables for the half of GDP. The share of investment is determined at 21 percent and of government consumption close to 19 percent. Net exports account for almost 4 percent.

Table 4.2: Long-run ratios in percent of GDP (1991–2018)

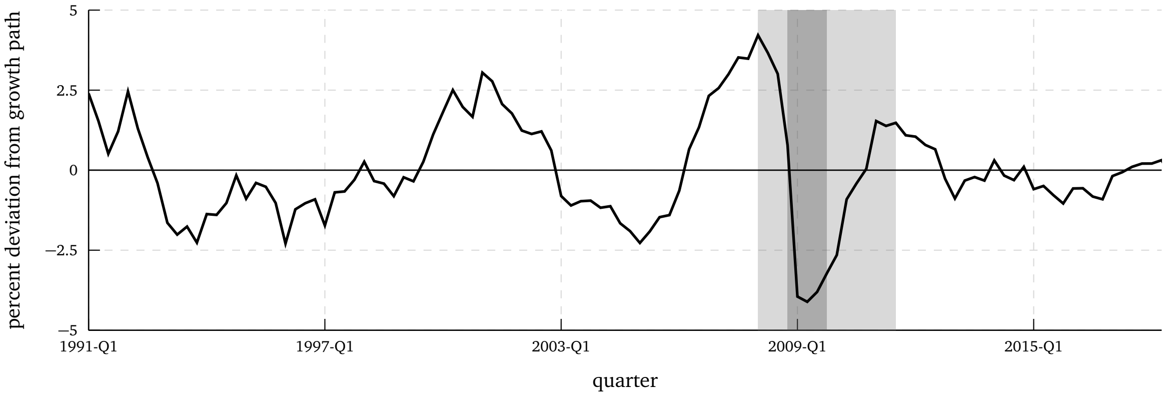
Description	$\overline{x_t/\text{GDP}_t}$
Private Consumption Expenditure	56.05
Non-durables consumption	49.72
Durable consumption	06.33
Investment	21.32
Government consumption.	18.87
Net exports	03.76

Source: See Appendix 4.C, own calculations.

Figures 4.2 and 4.3 present the cyclical behavior of GDP, its subaggregates and hours worked. The time series are the relative deviations from the concerning linear trend. We choose a linear trend filter instead of the commonly used HP-filter to be consistent with our estimation strategy.[5]

We observe a boom-bust cycle in GDP at about the same time of the dot-com bubble. This cycle was followed by a recovery from 2005 till 2008, which ended in a heavy drop. This drop depicts the Great Recession. GDP recovered fast and has moved along the long-run trend since then.

Figure 4.2: Cyclical behavior of GDP



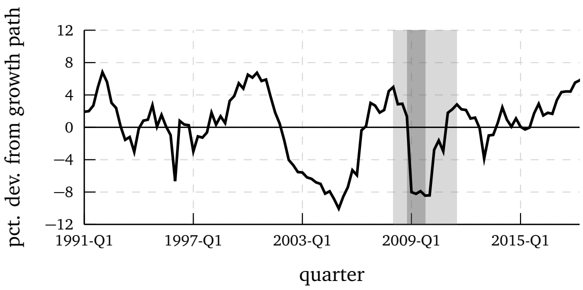
Notes: The data is presented as relative deviations from linear trend. The light gray area indicates the crisis from 2008-Q1 – 2011-Q3, the dark gray area indicates the main effective period of the fiscal stimulus program 2008-Q4 – 2009-Q4. **Source:** see Appendix 4.C, own calculations.

Panel (a) of Figure 4.3 shows that investment has co-moved with GDP, but with a higher volatility. Panel (b) displays two heavy short boom-bust-cycles of durables. The first peaked at the end of 2006, shortly after the announcement of a value-added tax (VAT) increase. This was followed by a bust at the beginning of 2007, when the increase took place. We observe the second peak at the same time as the German cash for clunkers program, which was also followed by a bust as the program expired. Government consumption was above its trend in the middle and late 1990's. It decreased at the beginning of the 2000's and increased from 2008 till 2010. Since 2010 it has fluctuated around its trend. Non-durable consumption was below its trend in the aftermath of the reunification, and was above the trend in the 2000's until the Great Recession and decreased slightly afterwards. Net exports relative to GDP decreased from

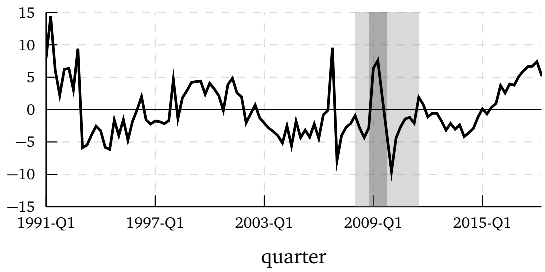
[5] Flor (2014) presents an overview of HP-filtered second moments of similar data.

1997 till 2001 from their trend, and increased sharply afterwards till 2003. From then on until the crisis they moved above the trend. Since the crisis they have fluctuated around the trend. In the medium-run, hours worked declined after the German reunification till 2005 and from then on they have increased. Hours worked have co-moved with GDP from 2000 onwards.

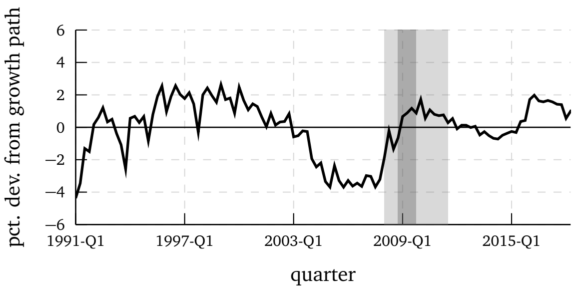
Figure 4.3: Cyclical behavior of different economic measures



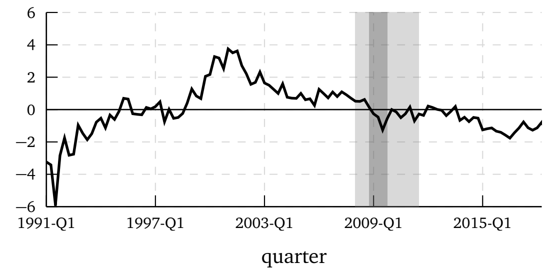
(a) Investment



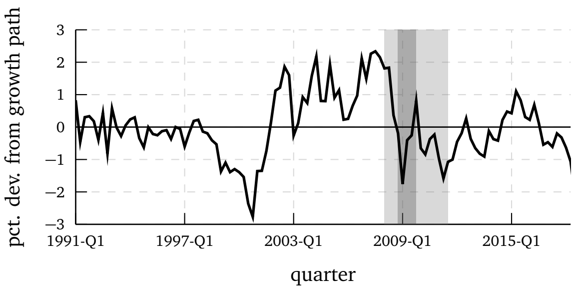
(b) Durables



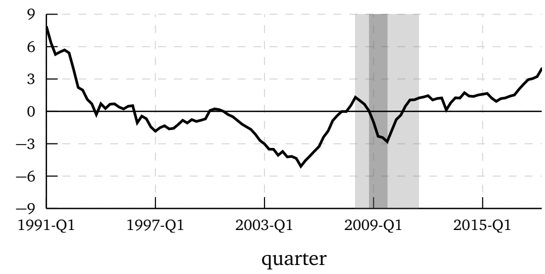
(c) Government Consumption



(d) Non-durable Consumption



(e) Net exports to GDP



(f) Hours

Notes: Despite hours worked, the data are presented as relative deviations from the corresponding linear trend. Hours worked is the relative deviation from the average. The light gray area indicates the crisis from 2008-Q1 – 2011-Q3, the dark gray area indicates the main effective period of the fiscal stimulus program 2008-Q4 – 2009-Q4. **Source:** see Appendix 4.C, own calculations.

The light gray area in Figures 4.2 and 4.3 indicates the Great Recession. GDP, hours worked and investment decreased from the end of 2008 until the peak of the crisis in 2009-Q2 by 5 percentage points, 4 percentage points and 12 percentage points, respectively. Their recovery completed in 2011. Durables increased during the time of the car subsidy – indicated through the dark gray area – by 12 percentage points and decreased by 18 percentage points afterwards. Durables recovered at the end of 2010. Government consumption increased at the beginning of 2009 by 5 percentage points and remained till the end of 2011 by 4 percentage points above its trend. Non-durables were less than 2 percent below their trend at the end of 2009 and recovered fast.

4.3 Methods

4.3.1 The prototype economy

The prototype economy consists of an infinitely-lived household, a firm facing perfect competition, and a government which finances its expenditures by levying taxes on labor, durables, and investment. The model of Chari, Kehoe, and McGrattan (2007) is extended in three ways. First, we distinguish between government spending and net exports and second, exclude durables from aggregated investment goods. Both enable a deeper analysis of the stimulus program and the former allows to account for the strong export-dependency of the German economy. Third, wedges consist of a growth and a business cycle part. This allows separate procedures for growth and business cycle accounting and ensures stationarity of the stochastic process. The model also accounts for productive capital and durable consumption capital adjustment costs. Chang (2000) shows that adjustment costs for capital goods in the market and at home solves problems with excess volatility and negative co-movements, because adjustment costs lower the substitutability, which is why we model this structural friction explicitly. The model is written in per capita terms.

4.3.1.1 Model

The per period utility of the representative household is parameterized as follows

$$u(C_t, D_t, N_t) = \begin{cases} \phi \ln(C_t) + (1-\phi)\ln(K_{Dt}) + \psi \ln(1-N_t) & \text{for } \eta = 1, \\ \frac{\left(C_t^{\phi} \cdot K_{Dt}^{1-\phi} \cdot (1-N_t)^{\psi}\right)^{1-\eta} - 1}{1-\eta} & \text{for } \eta \neq 1, \end{cases} \tag{4.3.1}$$

where C_t denotes consumption of non-durable goods and N_t is the household's labor supply. The stock of durable consumption goods K_{Dt} accumulates according to

$$\gamma_n K_{Dt+1} = (1-\delta_D)K_{Dt} + D_t - \Theta_{Dt}\left(\frac{D_t}{K_{Dt}}\right)K_{Dt}, \; \Theta_{Dt}\left(\frac{D_t}{K_{Dt}}\right) = \frac{a_D}{2}\left(\frac{D_t}{K_{Dt}} - b_D\right)^2, \tag{4.3.2}$$

where γ_n denotes the population growth factor, D_t are investments in durable consumption goods, and b_D is the ratio of investment in durables to the stock of durables in the long run. The household maximizes its expected life-time-utility

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \gamma_n)^s u(C_{t+s}, K_{Dt+s}, N_{t+s}) \tag{4.3.3}$$

subject to the budget constraint

$$C_t + (1+\tau_{It})P_{It}I_t + (1+\tau_{Dt})P_{Dt}D_t \leq R_t K_{It} + (1-\tau_{Nt})W_t N_t + T_t - P_{Et}E_t, \tag{4.3.4}$$

where K_{It} denotes the productive capital stock (capital stock hereafter), I_t investment in capital, T_t lump-sum transfers, E_t net exports, R_t the rental rate on capital, and W_t the real wage. The tax rates τ_{Nt}, τ_{It} and τ_{Dt} are used to model wedges in the labor, investment and durables market. P_{Et}, P_{It} and P_{Dt} are the relative prices for net exports, investment, and durable goods and reflects the wedges' long-run element. The consumption good is the numeraire. The capital stock follows the law-of-motion

$$\gamma_n K_{It+1} = (1-\delta_I)K_{It} + I_t - \Theta_{It}\left(\frac{I_t}{K_{It}}\right)K_{It}, \; \Theta_{It}\left(\frac{I_t}{K_{It}}\right) = \frac{a_I}{2}\left(\frac{I_t}{K_{It}} - b_I\right)^2, \tag{4.3.5}$$

with b_I as the investment-to-capital ratio in the long run.

The representative firm produces its output good Y_t with the Cobb-Douglas technology

$$Y_t = K_{It}^{\alpha}(\gamma_z^t Z_t N_t)^{1-\alpha} \tag{4.3.6}$$

and faces perfect competition. The parameter γ_z denotes the growth factor of labor augmenting technical progress and Z_t the efficiency wedge.

The government expenditures G_t are exogenous and the government chooses lump-sum transfers T_t, so that its budget constraint

$$P_{Gt}G_t + T_t \leq \tau_{Nt}W_tN_t + \tau_{It}P_{It}I_t + \tau_{Dt}P_{Dt}D_t \tag{4.3.7}$$

always binds. Thereby, the resource constraint of the economy is

$$Y_t = C_t + P_{It}I_t + P_{Dt}D_t + P_{Gt}G_t + P_{Et}E_t. \tag{4.3.8}$$

Growth component: As already mentioned, the population grows with γ_n and technical progress with γ_z. Furthermore, the wedges evolve differently. The relative prices reflect this. In the long run $P_{Xt} \in \{P_{It}, P_{Dt}, P_{Gt}, P_{Et}\}$ evolves with $P_{Xt} = g_{P_X} P_{Xt-1}$. The ensuing trend growth factors of different variables X_t are described in Table 4.3. These variables are scaled by $x_t = \frac{X_t}{g_X^t}$ and are thus stationary variables.

Table 4.3: Growth factors

X_t	Y_t	C_t	W_t	T_t	I_t	K_{It}	R_t	D_t	K_{Dt}	G_t	E_t	γ_z	N_t	P_{Xt}
g_X	g_Y	g_Y	g_Y	g_Y	g_I	g_I	g_Y/g_I	g_D	g_D	g_G	g_E	$g_Y^{\frac{1}{1-\alpha}} g_I^{\frac{\alpha}{\alpha-1}}$	1	$g_{P_X} = \frac{g_Y}{g_X}$

Business cycle component: The VAR(1)-process

$$\mathbf{s_{t+1}} = \Pi \mathbf{s_t} + \epsilon_{\mathbf{t+1}}, \; \epsilon_{\mathbf{t}} \sim \mathcal{N}(0, \Sigma), \tag{4.3.9}$$

drives the fluctuation of the model, where

$$\mathbf{s_t} = \begin{bmatrix} \ln(s_{At}) & s_{Nt} & s_{It} & s_{Dt} & s_{Et} & \ln(s_{Gt}) \end{bmatrix}^{\mathbf{T}},$$
$$\epsilon_{\mathbf{t}} = \begin{bmatrix} \epsilon_{At} & \epsilon_{Nt} & \epsilon_{It} & \epsilon_{Dt} & \epsilon_{Et} & \epsilon_{Gt} \end{bmatrix}^{\mathbf{T}}.$$

The stochastic process affects the wedges as follows

$$\begin{aligned} Z_t &= A^* \cdot s_{At}, & \tau_{Nt} &= \tau_N^* + s_{Nt}, & \tau_{It} &= \tau_I^* + s_{It}, \\ \tau_{Dt} &= \tau_D^* + s_{Dt}, & e_t &= e^* + s_{Et}, & g_t &= g^* \cdot s_{Gt}, \end{aligned}$$

where A^*, τ_N^*, τ_I^*, τ_D^*, e^* and g^* are the corresponding steady-state component of the different distortions. Similar to Chari, Kehoe, and McGrattan (2007), we define the six wedges as follows: The efficiency wedge Z_t, the net export wedge e_t, the government spending wedge g_t, the labor wedge $1 - \tau_{Nt}$, the investment wedge $\frac{1}{1+\tau_{It}}$, and the durables wedge $\frac{1}{1+\tau_{Dt}}$. The latter two are defined so that, similar to the labor market wedge, increases act like subsidies and decreases like taxes in comparison to the steady-state value. Since the cyclical component includes the steady-state component, detrended prices p_{Et}, p_{Gt}, p_{It}, p_{Dt} are normed to one. We present in Appendix 4.A the full dynamic equilibrium of the model with stationary variables.

Solution: To derive the model's decision rules, we use a linear perturbation method. In detail, we apply the method of undetermined coefficients as Uhlig (1999) and Christiano (2002) describe to solve the log-linearized model. The solved model then can be written as

$$\mathbf{y_t} = \mathbf{L_x^y} \cdot \mathbf{x_t} + \mathbf{L_s^y} \cdot \mathbf{s_t}, \tag{4.3.10}$$
$$\mathbf{c_t} = \mathbf{L_x^c} \cdot \mathbf{x_t} + \mathbf{L_s^c} \cdot \mathbf{s_t}, \tag{4.3.11}$$
$$\mathbf{x_{t+1}} = \mathbf{L_x^x} \cdot \mathbf{x_t} + \mathbf{L_s^x} \cdot \mathbf{s_t}, \tag{4.3.12}$$

where the matrices $\mathbf{L_x^{\cdot}}$ characterize the policy function of the deterministic part of the model's solution, while $\mathbf{L_s^{\cdot}}$ describe the policy function of the stochastic part. With $\hat{x}_t = \ln(x_t) - \ln(x)$ as the approximation of the relative deviation of a variable x_t from its steady state value x, the vector of observables is $\mathbf{y_t} = \begin{bmatrix} \hat{y}_t & \hat{N}_t & \hat{i}_t & \hat{d}_t & \hat{g}_t & \widehat{\frac{e_t}{y_t}} \end{bmatrix}^{\mathbf{T}}$, while $\mathbf{c_t}$ denotes the vector of unobserved control variables and $\mathbf{x_t} = \begin{bmatrix} \hat{k}_{It} & \hat{k}_{Dt} \end{bmatrix}^{\mathbf{T}}$ the vector of endogenous states.[6]

4.3.1.2 Mapping

Chari, Kehoe, and McGrattan (2007), Brinca, Chari, Kehoe, and McGrattan (2016), and various other authors map structural models into their prototype economy. Nutahara and Inaba (2012) apply BCA for misspecified wedges and find they are able to approximate the true wedges and the corresponding response of the agents adequately. We show first how to map the stimulus program to the prototype economy. Since the wedges' drivers are modeled as taxes, this is straightforward for most of the measures. Secondly, we reflect monetary policy.

Mapping the stimulus program

Government Wedge: We assign total government spending to the government spending wedge. These are mainly investments in infrastructure and financial support for local and state authority spending. Hence, the stimulus program increases the government wedge directly.

Durables Wedge: The two measures concerning new cars affect the durables wedge. For a given producer price, both measures reduce the absolute tax or the relative price of durables from the households perspective. Hence, they increase the durables wedge.

Investment Wedge: The first part of the stimulus program which affects the investment wedge are subsidies for investments in innovations. The second are increased tax deductions by allowing for a reducing-balance method. For given producer prices, absolute taxes or the relative price of investment decreases and thus the investment wedge increases.

Chari, Kehoe, and McGrattan (2007) show how to map financial frictions in terms of a financial accelerator and Brinca, Chari, Kehoe, and McGrattan (2016) show how to map financial frictions in terms of collateral constraints into a prototype economy with an investment wedge. The loan and guarantee program lowers financial frictions, in particular they mitigate the banks' collateral constraints. Following this, the loan and guarantee program also raises the investment wedge.

Labor Wedge: The stimulus program loweres income tax and social contribution, this increases the labor wedge in general.

Brinca, Chari, Kehoe, and McGrattan (2016) show the link between a prototype economy with efficiency and labor wedges and an economy with search and matching frictions. The mentioned labor market actions, e.g. expanded short-time work, reduce such frictions and thus, increase the labor market wedge. The effects should be delayed in time due to lower hiring frictions in the aftermath of the crisis.

[6] The use of $\widehat{\frac{e_t}{y_t}}$ instead of $\hat{e}_t$ is discussed in 4.3.2.2.

Efficiency Wedge: Due to the labor market actions in the previous paragraph, the efficiency wedge increases also due to a better matching. Further, the expanded short-time work possibilities reduce labor hoarding, since the firm can both retain employees to lower future hiring frictions and adjust hours worked. As a consequence, the efficiency wedge increases.

As shown by Chari, Kehoe, and McGrattan (2007), input-financing frictions are associated with efficiency wedges. These frictions appear when firms must borrow for an input good and some firms are financially more constrained than others. Such firms have to pay higher interest rates. The loan and guarantee program lowers financial constraints and thus increases the efficiency wedge.

Net exports: The increase in Hermes coverage advances the conditions for exports. Nevertheless, the effects are probably only rather small.

Mapping monetary policy

Government Wedge: Purchasing bonds lowers the bonds' interest rates and this lowers the costs of debt-financed government spending, which may indirectly increase the government wedge.

Durables Wedge: Since refinancing is cheaper, for a given real rate of return, investment increases. Hence, monetary policy changes the intertemporal decision of a household, which is reflected in a higher durables wedge. Furthermore, provision of liquidity also changes the intertemporal decisions of liquidity constrained households, which also reflects in a higher durables wedge.

Investment Wedge: Both mentioned effects of the durables wedge have the same effect on the investment wedge. The provision of liquidity and cheaper refinancing lowers frictions in the investment market.

As already mentioned, Brinca, Chari, Kehoe, and McGrattan (2016) show how to map an economy with a collateral constrained bank into a the prototype economy with an investment wedge. Lower collateral constraints lower frictions in the investment market. Thus, the slacked collateral requirements by the ECB increase the investment wedge.

Efficiency Wedge: As mentioned above, input-financing frictions are associated with efficiency wedges (see Chari, Kehoe, and McGrattan, 2007). The friction appears when firms must borrow for input goods and some firms are financially more constrained than others. Those firms have to pay higher interest rates. The Security Markets Program can lower these frictions and thus, increases the efficiency.

4.3.1.3 Calibration

We estimate the elasticity, $\eta_I = \frac{I}{K_I}\Phi_I''$, of the price of capital with respect to the investment to capital ratio as well as the elasticity, $\eta_D = \frac{D}{K_D}\Phi_D''$, of the price of the stock of durables with respect to the new durables to stock of durables ratio in addition to the parameters that characterize the stochastic process $\mathbf{s_t}$. The remaining parameters are calibrated as follows:

The capital elasticity α is set to 0.34. Flor (2014) calculates this as the German capital share from 1991 to 2012. In line with Heer and Maussner (2009, Chapter 1.5), Flor (2014) also provides the discount parameter $\beta = 0.994$ for the German economy. We pin down the annual rate of capital depreciation at the average ratio of gross fixed capital formation and the net stock of fixed assets. The average quarterly capital depreciation rate arises from $\delta_I = 1-(1-\delta_{I,annual})^{\frac{1}{4}}$. In the same manner the rate of durables depreciation δ_D is computed.

The choice of ψ, ϕ and η, which characterize the household's preferences, is more problematic. For ψ and η we follow the baseline calibration from Chari, Kehoe, and McGrattan (2007) and fix ψ at 2.24 and η at 1. We calibrate the preference weight of durables ϕ by matching the durable to non-durable consumption ratio with the long-run marginal rate of substitution between consumption and durables. We do not estimate the steady-state values of the different wedges.

Instead, we compute them from the model's static equilibrium equations in line with Lama (2011). We fix the steady-state values of output, government consumption, investment in capital as well as in durables to their average shares of output (see Table 4.2). The steady-state labor supply N is 0.122, which equals the average share of hours worked on the available time budget of a household.[7] Our calibration exercises are summarized in Table 4.4.

Table 4.4: Calibration of the model

Parameter	Description	Value
α	capital share	0.34
β	discount factor	0.994
δ_I	rate of capital depreciation	0.017
δ_D	rate of durables depreciation	0.045
ψ	preference weight of labor	2.24
ϕ	preference weight of consumption	0.879
η	risk aversion	1

4.3.1.4 Identification

We check our prototype economy for strict local identification following Iskrev (2010), who shows that a linearized DSGE model with normally distributed shocks is locally identified for a given set of parameters, if the Jacobian matrix of theoretical first and second moments with respect to these parameters has full rank. To check the identifiability over a sufficiently large parameter space we draw 1,000,000 times from the following distributions for the elasticities of the adjustment costs η_D, η_I, for the the off-diagonals π_{ij}, $i \neq j$ of Π, for the diagonals π_{ii} of Π, and the elements b_{ij}, $i \leq j$ of the lower triangular matrix B with $\Sigma = BB^T$:

$$\eta_D, \eta_I \sim U(0,4), \qquad \pi_{ij} \sim \mathcal{N}(0,0.1), \qquad \pi_{ii} \sim \mathcal{N}(0.8,0.1), \qquad b_{ij} \sim U(-0.05,0.05).$$

The Jacobian of the first and second moments (up to two lags) has full rank at approximately 99.9 percent of the draws. Thus, the model is virtually identifiable in the chosen parameter space.[8]

Brinca, Iskrev, and Loria (2018) provide and apply strategies for identification strength. They show that weak identification of the stochastic process' parameters is secondary, but this does not hold for structural ones. To address this problem, we compute the likelihood surface of the uncertain deep parameters η_D and η_I to detect a global maximum as well as the likelihood's curvature and execute robustness checks in section 4.4.

4.3.2 The business cycle accounting procedure

The BCA procedure is divided into three separate steps: The estimation of the parameters, the identification of the wedge states, and the assessment of the contribution of a single wedges towards the business cycle.

MLE determines the matrices Π and Σ that characterize the stochastic process $\mathbf{s_t}$ as well as the elasticities η_I and η_D that define the level of adjustment costs. Full-information estimation of DSGE models is typically done with Bayesian methods, although MLE involves less

[7]Here we follow (Heer and Maussner, 2009, Chapter 1.5), who assume that the household's maximum working hours amount to $1,440 = 16$ hours per day$\times$90 days per quarter.

[8]In comparison, we proceed similarly for the benchmark economy of Chari, Kehoe, and McGrattan (2007) presented in Appendix 4.B. The Jacobian of the first and second moments (up to two lags) has no full rank at 26 parameter draws from 1,000,000.

assumptions. Applying Bayesian estimation is usually meaningful, since the researcher has a structural parametrization in mind and, by association, an idea of probable parameter values. We would like to stress that the application of BCA requires MLE and any restrictions like the Bayesian approaches, such as Otsu (2010), Chakraborty and Otsu (2013) or Plotnikov (2017) are questionable. The wedges are superpositions and interactions of a variety of market distortions with an underlying reduced-form stochastic process, which complicates the interpretation of the Markov transitions. Furthermore, recall the findings of Nutahara and Inaba (2012) that the VAR(1) strips a potentially more sophisticated stochastic process down. Thus, the estimated parameters are only pseudo-true for the real model. As a consequence, in general the values of the process' parameters cannot be interpreted, and a-priori assumptions of them are meaningless, and even more seriously, may restrict the set of mappable models. Thus, we make a point for MLE and let the data speak through an unrestricted VAR.[9]

After all parameters are pinned down, either by calibration or MLE, we use a state-smoothing algorithm as described in Durbin and Koopman (2012, Chapter 4.4) to predict the wedge's states $\mathbf{s_t}$.

In a last step, in line with Chari, Kehoe, and McGrattan (2007), we feed the wedges separately back into the model, while others are set constant, to assess the contribution of each wedge to the quantities of interest.[10]

4.3.2.1 MLE

To evaluate the likelihood function of the linear state-space model (4.3.9)–(4.3.12), most of the literature uses a Kalman-recursion initialized at the unconditional mean and variance of the state vector $[\mathbf{x_0^T}\ \mathbf{s_0^T}]^{\mathbf{T}}$ (see e.g. DeJong and Dave, 2011, Chapter 8.4)). However, for an asymptotic stable state-space model, the mean squared error (MSE) $\mathbf{P_{t|t}}$ of the point estimate for $[\mathbf{x_t^T}\ \mathbf{s_t^T}]^{\mathbf{T}}$ conditional on a observed set of data $\{\mathbf{y_1}, \dots, \mathbf{y_t}\}$ converges to a matrix $\mathbf{P}$, the steady-state MSE, as t goes to infinity.[11] Exploiting this property, Chari, Kehoe, and McGrattan (2007) use the steady-state MSE $\mathbf{P}$ instead of the unconditional variance to initialize their Kalman-recursion. As pointed out by Huber (2020), it can be shown that the steady-state MSE $\mathbf{P}$ is equal zero in standard DSGE models like the one presented here.[12] To get the intuition behind the result and for the sake of simplicity, let us consider the case without growth and with zero adjustment costs. In this case, equations (4.3.2) and (4.3.5) rewrite to

$$\begin{aligned} K_{Xt+1} &= X_t + (1-\delta_X)K_{Xt} \\ &= \sum_{i=0}^{t-1}(1-\delta_X)^i X_{t-i} + (1-\delta_X)^t K_{X1},\ X \in \{I, D\}. \end{aligned}$$

Imagine we observe the investment $X_i \in \{I, D\}$ in capital and in durables for all $i = 1, \dots, t$. Assuming that K_{X1} is normally distributed with variance σ_X^2, the variance of K_{Xt+1} conditional on $\{X_1, \dots, X_t\}$ yields $(1-\delta_X)^{2t}\sigma_X^2$. Since $\delta_I, \delta_D \in (0, 1]$, it is straightforward that the uncertainty regarding the endogenous states $\mathbf{x_t}$ disappears as t goes to infinity. Furthermore, assuming $\mathbf{L_s^y}$ is

[9] We would like to point out two technical issues regarding Bayesian methods and BCA. First, to the best of our knowledge, there is no prior that includes all combination parameter values that generate eigenvalues of Π less than one and excludes all combinations that do not have these properties. Second, the posteriors of a VAR-driven DSGE model can be multi-modal. This makes the commonly used RWMH algorithms unsuitable. For a deeper discussion and solution for the latter issue, see Herbst and Schorfheide (2015, Chapter 5, 6.1)

[10] See the technical appendix by Chari, Kehoe, and McGrattan (2007) for more details.

[11] For a formal proof, see e.g. Hamilton (1994, Chapter 13).

[12] As long as $\mathbf{L_s^y}$ is non-singular and $\frac{1-\delta_D}{\gamma_n \cdot g_D}, \frac{1-\delta_I}{\gamma_n \cdot g_I} \in [0, 1)$ our prototype economy satisfies the preconditions of Proposition 1 by Huber (2020).

non-singular,[13] it follows that

$$\mathbf{s_t} = \left[\mathbf{L_s^y}\right]^{-1}\left(\mathbf{y_t} - \mathbf{L_x^y}\cdot\mathbf{x_t}\right). \tag{4.3.1}$$

Thus, as the uncertainty of the endogenous states $\mathbf{x_t}$ disappears as t goes to infinity, the uncertainty over the exogenous states $\mathbf{s_t}$ disappears as well. Using a Kalman-recursion initialized at the steady-state, with the steady-state MSE **P** is therefore equivalent to the assumption that the initial state vector is fixed and known, $[\mathbf{x_0^T}\ \mathbf{s_0^T}]^\mathbf{T} = \mathbf{0}_{nx+ns\times 1}$. Huber (2020) elaborates two major advantages of a fixed and known initialization at the long-run equilibrium. First, the likelihood evaluation can be vectorized and more important, it provides an analytical solution of the MLE for Σ since we can observe the residuals ϵ_t independently of Σ.[14] The solution of the MLE for Σ for a given Π is

$$\hat{\Sigma} = \frac{\mathbf{1}}{\mathbf{N}}\sum_{\mathbf{t=1}}^{\mathbf{N}}\left[(\mathbf{s_t} - \Pi\cdot\mathbf{s_{t-1}})\cdot(\mathbf{s_t} - \Pi\cdot\mathbf{s_{t-1}})^\mathbf{T}\right],\ \mathbf{s_0} = \mathbf{0}_{ns\times 1}. \tag{4.3.2}$$

The estimates of a standard Kalman-recursion, which is initialized at the unconditional first and second moments, are more natural, since the initial states are usually unknown. Huber (2020) however shows that the average likelihood of the steady-state Kalman-recursion converges pointwise to the average likelihood of the standardly initialized Kalman-recursion. Therefore, we use the estimates of the steady-state Kalman-recursion as the initial guess for a second estimation, where we initialize the Kalman-recursion with the unconditional first and second moments.

4.3.2.2 Data manipulation

The observables are GDP, investment, durables, government expenditures, net exports to GDP, and hours worked. Regressions with the logarithm of the first four observables as dependent variable and time as independent variable provide necessary components. The coefficient estimates determine the growth rates and the residuals the relative deviation from the particular growth path. Negative values for net exports prevent logarithmization. A regression with net exports relative to GDP as dependent variable and time as independent variable provides auxiliary variables. The coefficient is the excess growth rate of net exports compared to GDP growth. The residuals are the deviation from the long-run net exports to GDP rate, which is computable in the model. The residuals of these regressions are used for business cycle accounting, the coefficients for growth accounting.

Since hours worked per capita do not include a trend, the relative deviations from the long-run average are used for business cycle accounting. Whereas growth accounting is of course not applicable in this manner.

For a detailed data source, see Appendix 4.C.

[13] Huber (2020) discusses how to deal with cases where $\mathbf{L_s^y}$ is singular. However, this case never occurred in our analysis.

[14] Huber (2020) presents a detailed and more general version, Monte Carlo studies and further applications of this approach.

4.4 Results

4.4.1 Growth accounting

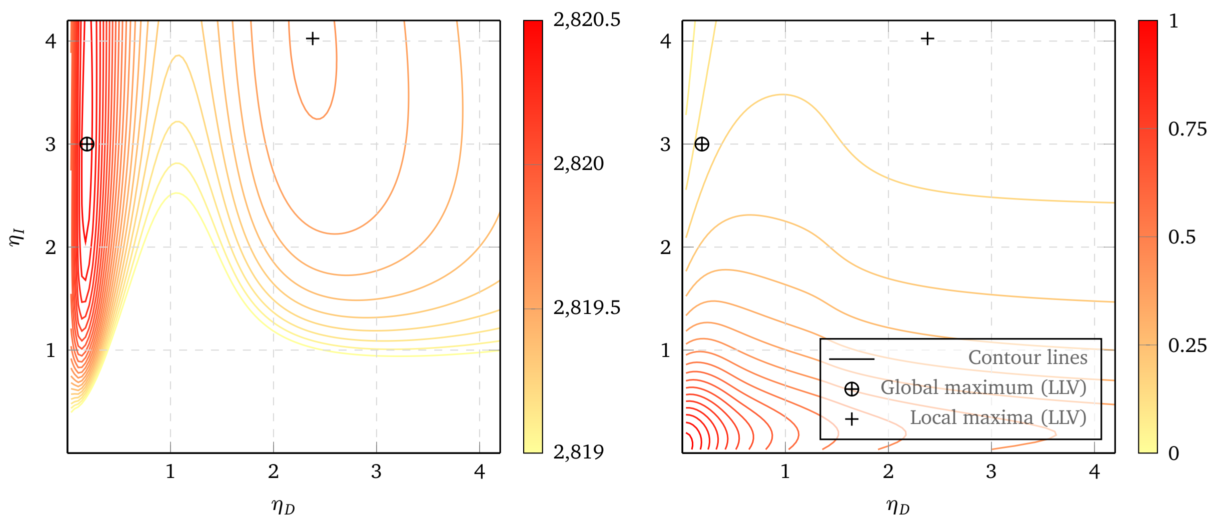
Table 4.5 presents the growth rates of the observables. The GDP annual trend growth rate is 1.32 percent. The amount of durables and investment goods grows slower than GDP, while net exports grow faster. Government consumption grows similar to GDP.

Table 4.5: Growth accounting

Parameter	Description	Value
$\ln(\gamma_n^4)$	annual growth rate of population	0.03%
$\ln(g_Y^4)$	annual growth rate of GDP	1.32%
$\ln(g_I^4)$	annual growth rate of investment	0.93%
$\ln(g_D^4)$	annual growth rate of durables	0.35%
$\ln(g_G^4)$	annual growth rate of gov. cons.	1.40%
$\ln(g_E^4)$	annual growth rate of net exports	1.65%

Similar to the shocks which drive the business cycle, the long-run components of the wedges P_{xt} and γ_z are reduced-form. Since we focus on the business cycle, we discuss only briefly potential causes for different growth rates. Differences in the long-run component of the durables and the investment wedge (P_{Dt}, P_{It}) may occur due to investment-specific technological change as described by Greenwood, Hercowitz, and Krusell (1997). The increase in German net exports since the launch of the Euro is investigated by in't Veld, Vogel, Ratto, Kollmann, and Roeger (2014). The most important factors, summed up in P_{Et}, are: A higher German savings rate, positive supply shocks, especially due to labor market reforms, as well as a higher demand for German goods of non Euro area members.

Figure 4.4: Maximum-Likelihood-Estimation



(a) Log-Likelihood (without constant)

(b) Correlation between ϵ_{It} and ϵ_{Dt}

Table 4.6: Estimation of exogenous shock process

Autoregressive Matrix							
Π	$ln(s_A)$	s_N	s_I	s_D	s_E	$ln(s_G)$	
$ln(s_A)$	0.90	0.41	0.00	0.07	-0.21	-0.16	
s_N	0.01	0.83	0.01	-0.02	-0.12	-.01	
s_I	0.70	-1.71	0.96	-0.52	1.44	1.07	
s_D	0.27	-0.05	-0.00	0.66	0.16	-0.01	
s_E	0.06	-0.03	0.01	-0.05	0.62	-0.12	
$ln(s_G)$	-0.05	0.17	-0.01	-0.05	-0.22	0.80	

Correlation and standard errors							
$Corr(\epsilon_i, \epsilon_j)$	ϵ_A	ϵ_N	ϵ_I	ϵ_D	ϵ_E	ϵ_G	$100 \cdot StD(\epsilon_i)$
ϵ_A	1.00						0.94
ϵ_N	0.03	1.00					0.34
ϵ_I	-0.49	-0.06	1.00				7.12
ϵ_D	0.27	-0.83	0.13	1.00			1.44
ϵ_E	0.31	0.70	-0.02	-0.36	1.00		0.59
ϵ_G	-0.10	0.13	-0.19	-0.16	-0.13	1.00	0.80

4.4.2 Estimation

As already mentioned, the MLE includes Π, Σ, η_D and η_I. Panel 4.4(a) illustrates the likelihood function with respect to η_D and η_I, while Π and Σ are the argument maximum of the function for given η_I and η_D. The panel identifies two local maxima. The global is at $\eta_D = 0.19$ and $\eta_I = 3.00$.

Table 4.6 presents the estimates for the autoregressive matrix Π as well as second moments of the innovations ϵ_i. All wedges are highly autoregressive. The investment wedge depends heavily on the other wedges with one lag. The innovations of the investment wedge have the highest volatility and are negatively correlated with the efficiency wedge. There is also a strong negative correlation between the innovations of the durables and the labor wedge. The net export wedge's innovation correlates with the labor wedge.

Panel 4.4(b) illustrates that the innovations of durables and investments are perfectly correlated in the absence of adjustment costs. Fehrle (2019) investigates different investment goods, vector-autoregressive processes and adjustment costs in detail and argues that adjustment costs can be viewed as a underpinning mechanism of reduced-form correlated shocks. Here, e.g. the mentioned high substitutability between durables and investments is prevented either by perfect correlated innovations, adjustment costs or a nest of them. Hence, it is useless to separate investments and durables without adjustment costs, since the corresponding wedges must co-move. Otherwise, as a result of Chang (2000), the high substitutability would lead to an excessive volatility of durables and investments and negative co-movements between them. However, this is contradicted by the data.

4.4.3 Business Cycle Accounting for the Great Recession and the German fiscal stimulus program, 2008-Q1 – 2011-Q3

The graphical analysis of our BCA exercise is reported in Figure 4.5. In Panels 4.5(a) to 4.5(e) we confront the observations of GDP, its subaggregates and hours worked with the model's prediction when only one wedge is allowed to fluctuate.

Panel 4.5(a) illustrates that the crisis was mainly driven by the efficiency wedge. The investment and net exports wedge also contributed to the crisis. These three wedges together induced the decrease in GDP. The labor wedge contributed to the crisis from 2009-Q2 to 2009-Q4. Before, the wedge was counter-cyclical and afterwards it introduced the recovery. The durables wedge and government consumption were anti-cyclical. Panel 4.5(b) illustrates that the investment wedge drove the decline in investment mostly, while the efficiency wedge mattered little. The efficiency wedge influenced durables negatively as Panel 4.5(c) shows. The durables wedge on its own increased durables up to almost 50 percent in 2009. Afterwards, the wedge only had a slight impact. Panel 4.5(d) indicates that the efficiency wedge caused the decline in non-durable consumption mostly and the labor wedge partly. The durables and government consumption wedge had little impact on non-durable consumption. Panel 4.5(e) predicts the decline in net exports to GDP and the investment wedge introduced the decline in hours worked. The labor market wedge drove the decline between 2009-Q2 and 2009-Q4. Besides, the labor wedge was counter-cyclical. The other wedges were counter-cyclical. Theory teaches us that the wedges of both investment goods D_t and I_t react similar to monetary policy and financial frictions in general.[15] Thus, Chari, Kehoe, and McGrattan (2007) and many others aggregate them. The business investment wedge drove the decline in business investment during the crisis. Financial frictions and other distortions dominated the fiscal and monetary policy measures. This is not true for durables. The only appreciable difference between the wedges during the crises were the car subsidies. Further, the positive impact of the durables wedge occurred simultaneously with the subsidies. The wedge began to stimulate the demand of durable goods with the introduction of the tax exemption for new cars in 2008-Q4. In 2009-Q1 the cash for clunkers program started, while the stimulating effect increased strongly. The stimulus disappeared between 2009-Q4 and 2010-Q1 while the last pay-off took place in 2009-Q4. Hence, we attribute the large increase due to the durables wedge to the car subsidies and can map changes due to the durables as well as government spending wedge to the fiscal stimulus program. The measures in other markets are dominated by frictions. Thus, it is unfortunately impossible to give statements about the measures with the chosen method.

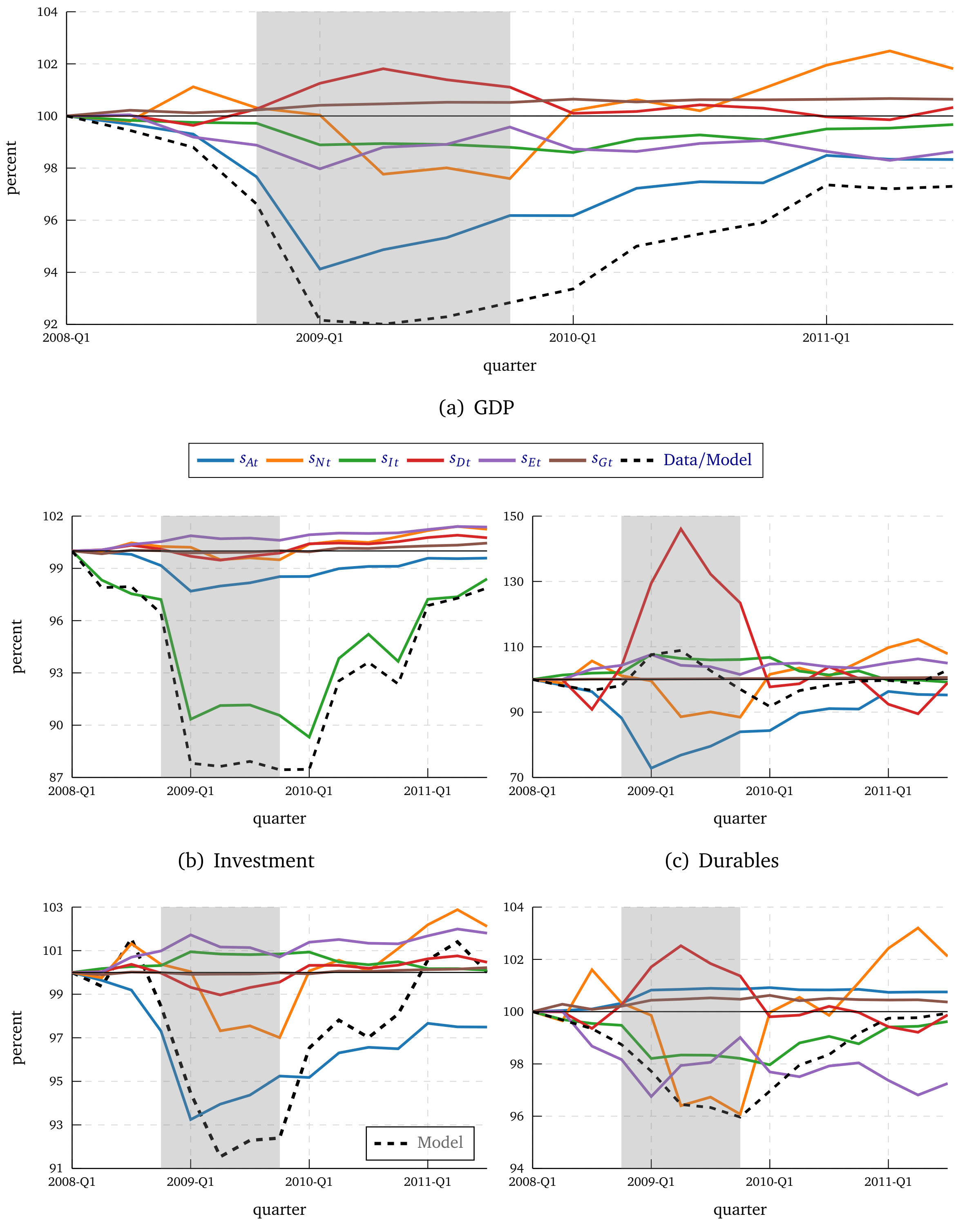
With respect to GDP and hours, we find that the stimulus program due to the durables subsidies and government consumption had a positive effect during the crisis. The model predicts an approximately 2 percent bigger decline in GDP and an approximately 3.5 percent bigger decline in hours without changes in those wedges during the peak of the crisis (2009-Q2). Regarding non-durable consumption and investment the effect of the stimulus program is negative. Nevertheless, during the crisis the stimulus of durables and government consumption increased GDP and was not completely substituted by lower investments and non-durable consumption. Intertemporal substitution of durables investment in the aftermath of the program was small. The bust was driven by the efficiency wedge, which depressed durables over the whole period. The durables wedge virtually did not influence GDP negatively from 2008-Q1 till 2011-Q3.

The labor market wedge mitigated the crisis at the beginning and the end of the crisis. In particular at the end of the crisis, the model predicts an increase of more than 2 percent in GDP and more than 3 percent in hours worked.

The measurement ω_i quantifies the contribution of each wedge to GDP during the Great

[15] Gertler and Gilchrist (2018) report for the U.S. financial frictions during the Great Recession a big negative impact on the durables market. Benmelech, Meisenzahl, and Ramcharan (2017) explain one third of the decline in the U.S. car demand by frictions on the asset-backed commercial paper market. The decline in U.S. house prices weakens the household balance sheets, which also had a negative effect on the U.S. auto market, as shown by Mian, Rao, and Sufi (2013).

Figure 4.5: BCA - Results



(a) GDP

(b) Investment

(c) Durables

(d) Non-durable Consumption

(e) Hours

Notes: Dashed lines for GDP, investment, durables and hours are the data and the model's outcome. Here they are equivalent. The dashed lines for non-durable consumption is only the model's outcome. The gray area indicates the main effective period of the fiscal stimulus program 2008-Q4 – 2009-Q4.

Recession as

$$\omega_i = \frac{\sum_t (\hat{y}_t^{GDP} - \hat{y}_t^i)}{\sum_j \sum_t (\hat{y}_t^{GDP} - \hat{y}_t^j)} \text{ with } i, j \in \{s_A, s_N, s_I, s_D, s_G, s_E\},\ t \in [2008\text{-}Q1, \ldots, 2011\text{-}Q3],$$

where $\hat{y}_t^{GDP}$ is the GDP when all wedges are non-changing and $\hat{y}_t^i$ is the model outcome of wedge i alone. Thus, the contribution of all wedges together sums to 1, while the sign of ω_i points out if wedge i has mitigated $(-)$ or amplified $(+)$ a crisis.

The efficiency wedge accounts for 62 percent of the decline in GDP during this period, net exports for 26 percent, the investment wedge for 19 percent, and the labor market accounts for 3 percent. Government consumption accounts for -5 percent and the durables wedge for -4 percent. Since the effect of the durables wedge during the durables subsidies was at least twice as large as the effect of government consumption and effects throughout the whole crisis were similar but expenditures for these subsidies only made up for about 25 percent of the increase of government consumption, durables subsidies were more efficient to stimulate aggregated demand than government consumption.

With the identifying assumption that the fiscal stimulus program together with monetary policy were the only counter-cyclical distortions, our results represent a lower bound for the impact of fiscal and monetary policy measures as well as for the pro-cyclical distortions.

4.4.4 Robustness and discussion

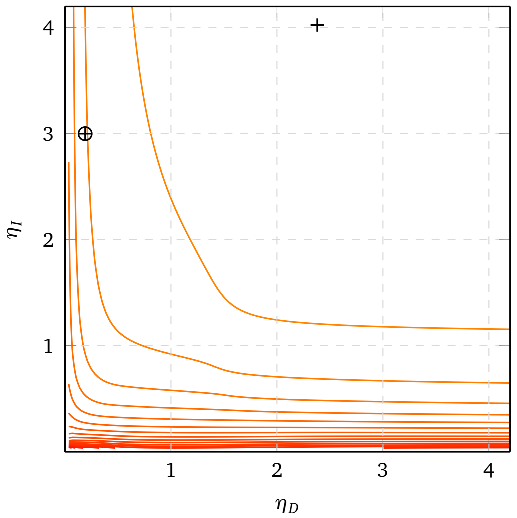
Robustness in parameters. The results depend potentially on the values of adjustment costs η_I, η_D and on the intertemporal elasticity of substitution η. To evaluate the sensitivity, we calculate ω_i over a grid of the mentioned parameters. Therefore, we reestimate the (remaining) uncertain parameters at each node of the parameter grid.

Figure 4.6 illustrates the contribution of the concerning wedges for different amounts of adjustment costs. The efficiency wedge contributed the most to the decline in GDP, followed by net export for the whole set of adjustment costs. The results for the labor market wedge and government consumption are robust as well. The durables wedge mitigated the crisis for most of the parameter combinations. The contribution would have been pro-cyclical without adjustment costs. As mentioned above, in the absence of adjustment costs a separation of the durables and investment wedge is meaningless. The investment wedge's contribution to the crisis would have been negative for $\eta_I < 1/3$ where the likelihood is the lowest (see Panel 4.4(a)) and positive otherwise.

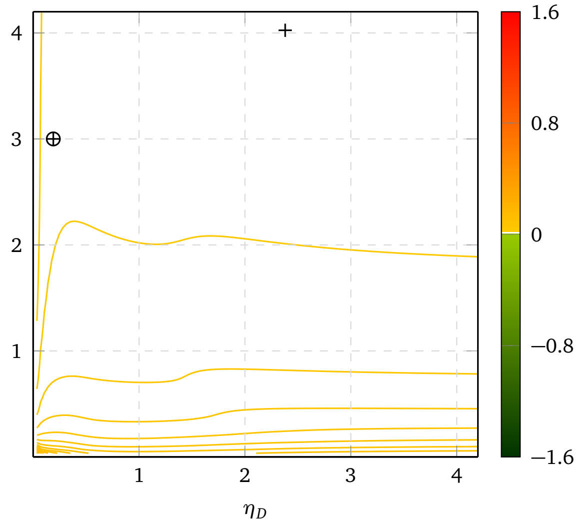
Subsidies in durables change the intertemporal rate of substitution. Hence, a robustness check to the elasticity of the substitution rate is relevant. Figure 4.7 presents the contribution to the decline in GDP over η. The contributions of the labor, investment, durables and the government consumption wedge are nearly constant. The contribution of net exports declines with a higher elasticity, nevertheless they contributed the second most over the whole domain. The contribution of the efficiency wedge increases with η.

Robustness regarding the benchmark model. The assessment of the joint contribution of the investment and durables wedge as well as the joint contribution of government consumption and net exports maps our economy into the benchmark BCA economy ex post. The left panel of Figure 4.8 illustrates these effects. The right panel plots the impact of the investment and government spending wedge in the Chari, Kehoe, and McGrattan (2007) benchmark economy, where durables and investment as well as government spending and net exports are aggregated

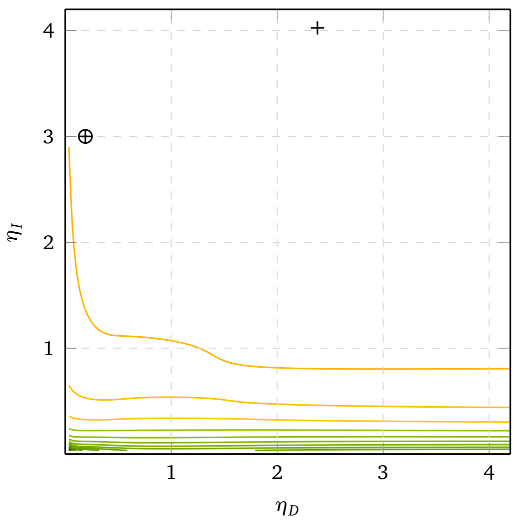
Figure 4.6: Adjustment costs specific wedge contribution



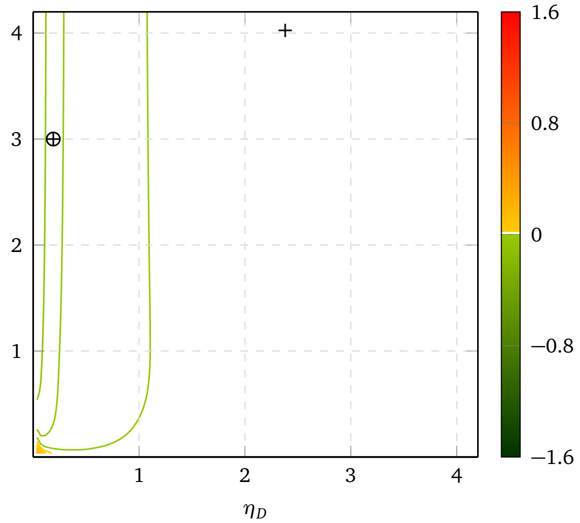
(a) Contribution of s_{At} to GDP ($\omega_{s_{At}}$)



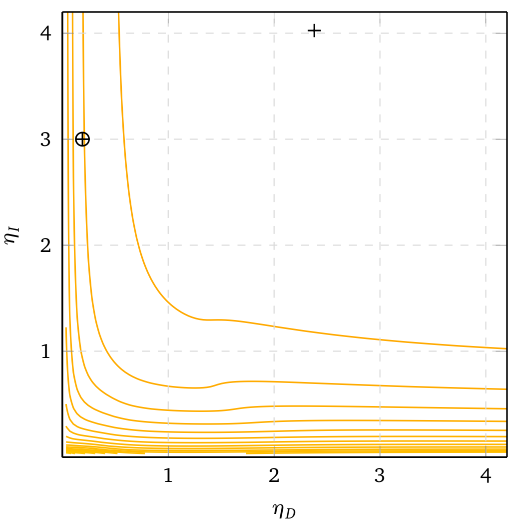
(b) Contribution of s_{Nt} to GDP ($\omega_{s_{Nt}}$)



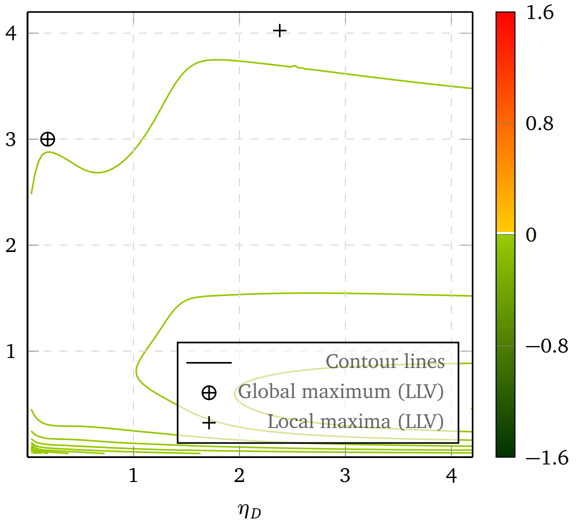
(c) Contribution of s_{It} to GDP ($\omega_{s_{It}}$)



(d) Contribution of s_{Dt} to GDP ($\omega_{s_{Dt}}$)

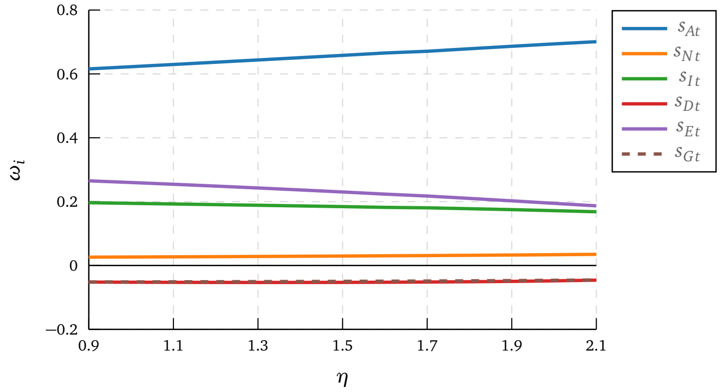


(e) Contribution of s_{Et} to GDP ($\omega_{s_{Et}}$)



(f) Contribution of s_{Gt} to GDP ($\omega_{s_{Gt}}$)

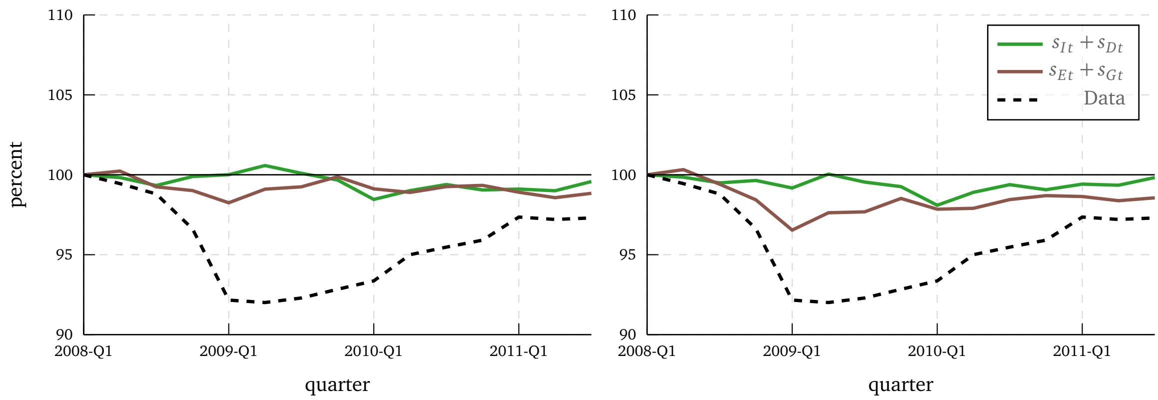
Figure 4.7: Inverse elasticity of intertemporal substitution specific wedge contribution



ex ante.[16] The results are similar, except in the more detailed economy the investment wedge was slightly counter-cyclical during the cash for clunkers program. Thus, the results of the detailed model are not counterfactual to the benchmark BCA model, but provide deeper insights.

Although the impact of the composed investment wedge was negligible during the Great Recession, our results suggest that the decomposed wedges were not. The pro-cyclical effect of the investment wedge and the policy-driven counter-cyclical effect of durables wedge offset each other. Hence, without our decomposition the importance of the investment wedge and, by association, the importance of financial frictions during the Great Recession is underrated. For example, the financial frictions of Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), or Gertler and Kiyotaki (2010) are equivalent to the investment wedge.

Figure 4.8: Robustness to the Chari, Kehoe, and McGrattan (2007) benchmark economy



(a) Detailed Economy

(b) Chari, Kehoe, and McGrattan (2007) - Benchmark Economy

Comparing two durables boom-bust cycles. As mentioned, there were two boom-bust cycles in the durables market. We compare them in Figure 4.9. Panels 4.9(a) and 4.9(b) show the data and the impact of the durables wedge on durables from 2008-Q1 to 2010-Q4 and from 2006-Q1 to 2007-Q4. The durables wedge accounts during the car subsidies programs for the boom, but only marginally for the bust afterwards. During 2006 a VAT increase announcement passed the institutions and at this time durables investments increased. The introduction of the increase

[16] Appendix 4.B sketches the model and provides our estimation strategy and results for the Chari, Kehoe, and McGrattan (2007) benchmark economy of the presented time series.

was in 2007-Q1, when the bust took place. The durables wedge caused the whole boom-bust cycle and illustrates intratemporal substitution.

Figure 4.9: The durables boom-bust cycles 2008-2010 and 2006-2007 in comparison

(a) Cash for clunkers program (b) VAT increase with announcement

4.5 Conclusion

We use the BCA analysis to investigate the impact of the German stimulus program during the Great Recession from 2008-Q1 to 2011-Q3. We extended the prototype economy by two wedges. Wedges correspond to the following variables: *government consumption, durables, investment, labor, net exports, and efficiency*. To account for the fiscal stimulus we map fiscal and monetary policy towards these wedges, thus enabling a policy evaluation.

We introduce two procedures that enable a fast and reliable MLE and the application of tools which help to overcome problems of weak identification. The first procedure separates between growth and business cycle accounting which ensures the stationarity of the underlying stochastic process. The second procedure is a new strategy to find a good guess for the argument maximum of the likelihood function. The applicability of MLE is crucial for, and one of the major advantages of BCA at the same time. Since MLE is difficult, and so Bayesian methods or other restrictions towards the stochastic process are used for BCA, we hope to give new impetus to the use of MLE and BCA with both procedures.

In our BCA analysis we find that the Great Recession in Germany was mainly driven by the efficiency wedge, net exports, and the investment wedge. In contrast, the durables and the government spending wedge acted counter-cyclical. We argue that the latter two collect parts of the German stimulus. The labor market wedge was pro-cyclical between 2009Q2 and 2009-Q4, besides it mitigated the crisis and especially induced the recovery. Due to higher expenditures for government consumption and a similar impact compared to the cash for clunkers program, subsidies for durable goods stimulated aggregated demand more efficiently. We check the robustness of our results to different choices of parameters that determine the elasticity of intertemporal substitution as well as capital and durables adjustment costs. We find that our results are robust for all wedges except the investment wedge. However, the results indicate that previous studies underrate the negative impact of the investment wedge and, as a consequence, the role of investment wedge equivalent financial frictions. We have to mention that BCA is only a first but useful step for the identification of market distortions, and thus we aim to motivate further research on the efficiency of durable goods' subsidies, the role of financial frictions during the Great Recession and the labor market driven recovery in Germany.

Bibliography

Benmelech, E., R. R. Meisenzahl, and R. Ramcharan (2017): "The Real Effects of Liquidity During the Financial Crisis: Evidence from Automobiles," *The Quarterly Journal of Economics*, 132(1), 317-365.

Berger, D., and J. Vavra (2015): "Consumption Dynamics During Recessions," *Econometrica*, 83(1), 101–154.

Bernanke, B., M. Gertler, and S. Gilchrist (1999): *Handbook of Macroeconomics*vol. 1C, chap. The Financial Accelerator in a Quantitative Business Cycle Framework, 1341–1393. North-Holland.

Brinca, P., V. Chari, P. Kehoe, and E. McGrattan (2016): "Accounting for Business Cycles," vol. 2, chap. Chapter 13, 1013–1063. Elsevier.

Brinca, P., N. Iskrev, and F. Loria (2018): "On Identification Issues in Business Cycle Accounting Models," .

Bundesgesetzblatt (2008): "Gesetz zur Umsetzung steuerrechtlicher Regelungen des Maßnahmenpakets "Beschäftigungssicherung durch Wachstumsstärkung".," Discussion Paper Nr. 64, Bonn, Teil I.

——— (2009): "Gesetz zur Sicherung von Beschäftigung und Stabilität in Deutschland," Discussion Paper No. 11, Bonn, Teil I.

Carlstrom, C. T., and T. S. Fuerst (1997): "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *The American Economic Review*, 87(5), 893–910.

Chakraborty, S., and K. Otsu (2013): "Business cycle accounting of the BRIC economies," *The B.E. Journal of Macroeconomics*, 13(1), 381–413.

Chang, Y. (2000): "Comovement, excess volatility, and home production," *Journal of Monetary Economics*, 46(2), 385 – 396.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2007): "Business Cycle Accounting," *Econometrica*, 75(3), 781–836.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2009): "New Keynesian Models: Not Yet Useful for Policy Analysis," *American Economic Journal: Macroeconomics*, 1(1), 242–266.

Cho, D., and A. Doblas-Madrid (2013): "Business cycle accounting East and West: Asian finance and the investment wedge," *Review of Economic Dynamics*, 16(4), 724–744.

Christiano, L. J. (2002): "Solving Dynamic Equilibrium Models by a Method of Undetermined Coefficients," *Computational Economics*, 20(1), 21–55.

DeJong, D. N., and C. Dave (2011): *Structural macroeconometrics*. Princeton Univ. Press, Princeton, NJ [et al.], 2. ed.

Drygalla, A., O. Holtemoeller, and K. Kiesel (2018): "The effects of fiscal policy in an estimated DSGE Model – the case of the German stimulus packages during the great recession," *Macroeconomic Dynamics*, first view, published online 22 nov., 1–31.

Durbin, J., and S. Koopman (2012): *Time Series Analysis by State Space Methods: Second Edition*, Oxford Statistical Science Series. OUP Oxford.

EUROPEAN CENTRAL BANK (2010): "Chronology of monetary policy measures of the Eurosystem," *Monthly Bulletin*.

——— (2011): "Chronology of monetary policy measures of the Eurosystem," *Monthly Bulletin*.

FEHRLE, D. (2019): "Housing and the business cycle revisited," *Journal of Economic Dynamics and Control*, 99, 103–115.

FLOR, M. (2014): "Post reunification economic fluctuations in Germany: a real business cycle interpretation," *Review of Business and Economics Studies*, 4, 5–15.

GADATSCH, N., K. HAUZENBERGER, AND N. STÄHLER (2016): "Fiscal policy during the crisis: A look on Germany and the Euro area with GEAR," *Economic Modelling*, 52, 997–1016.

GEHRKE, B., W. LECHTHALER, AND C. MERKL (2019): "The German labor market during the Great Recession: Shocks and institutions," *Economic Modelling*, 78, 192–208.

GERTH, F., AND K. OTSU (2018): "The post-crisis slump in Europe: a business cycle accounting analysis," *The BE Journal of Macroeconomics*, 18(1), 1–25.

GERTLER, M., AND S. GILCHRIST (2018): "What Happened: Financial Factors in the Great Recession," *Journal of Economic Perspectives*, 32(3), 3–30.

GERTLER, M., AND N. KIYOTAKI (2010): "Financial Intermediation and Credit Policy in Business Cycle Analysis," in *Handbook of Monetary Economics*, ed. by B. M. Friedman, and M. Woodford, vol. 3, chap. 11, pp. 547–599. Elsevier, 1st ed.

GREENWOOD, J., Z. HERCOWITZ, AND P. KRUSELL (1997): "Long-Run Implications of Investment-Specific Technological Change," *The American Economic Review*, 87(3), 342–362.

HAMILTON, J. D. (1994): *Time Series Analysis*. Princeton University Press.

HANSEN, G. D., AND L. E. OHANIAN (2016): "Neoclassical Models in Macroeconomics," *NBER Working Papers*, 22122, 1–97.

HEBOUS, S. (2010): "The effects of discretionary fiscal policy on macroeconomic aggregates: a reappraisal," *Journal of Economic Surveys*, 25(4), 674–707.

HEER, B., AND A. MAUSSNER (2009): *Dynamic General Equilibrium Modeling: Computational Methods and Applications*. Springer Berlin Heidelberg.

HERBST, E. P., AND F. SCHORFHEIDE (2015): *Bayesian estimation of DSGE models*. Princeton University Press.

HUBER, J. (2020): "Likelihood Evaluation of DSGE Models without Measurement Errors," unpublished memo, University of Augsburg.

IN'T VELD, J., L. VOGEL, M. RATTO, R. KOLLMANN, AND W. ROEGER (2014): "What drives the German current account? And how does it affect other EU Member States?," *Economic Policy*, 30(81), 47–93.

ISKREV, N. (2010): "Local identification in DSGE models," *Journal of Monetary Economics*, 57(2), 189–202.

KARABARBOUNIS, L. (2014): "The labor wedge: MRS vs. MPN," *Review of Economic Dynamics*, 17(2), 206–223.

KEHOE, T. J., AND E. C. PRESCOTT (eds.) (2002): *Great Depressions of the 20th Century*, vol. 5 (1) of *Review of Economic Dynamics*.

KERSTING, E. K. (2008): "The 1980s recession in the UK: A business cycle accounting perspective," *Review of Economic Dynamics*, 11(1), 179–191.

KIYOTAKI, N., AND J. MOORE (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–248.

LAMA, R. (2011): "Accounting for output drops in Latin America," *Review of Economic Dynamics*, 14(2), 295–316.

LEUWER, D., AND B. SÜSSMUTH (2018): "Assessing Temporary Product-Specific Subsidies: A Time Series Intervention Analysis," *CESifo Working Paper*, 6946, 1–45.

LU, S.-S. (2012): "East Asian growth experience revisited from the perspective of a neoclassical model," *Review of Economic Dynamics*, 15(3), 359–376.

MIAN, A., K. RAO, AND A. SUFI (2013): "Household Balance Sheets, Consumption, and the Economic Slump," *The Quarterly Journal of Economics*, 128(4), 1687–1726.

MIAN, A., AND A. SUFI (2012): "The effects of fiscal stimulus: Evidence from the 2009 cash for clunkers program," *The Quarterly Journal of Economics*, 127(3), 1107–1142.

MULLIGAN, C. B. (2005): "Public policies as specification errors," *Review of Economic Dynamics*, 8(4), 902–926.

NUTAHARA, K., AND M. INABA (2012): "An application of business cycle accounting with misspecified wedges," *Review of Economic Dynamics*, 15(2), 265–269.

OECD (2009): *OECD Economic Outlook, Volume 2009 Issue 1*.

OHANIAN, L. E. (2010): "The Economic Crisis from a Neoclassical Perspective," *Journal of Economic Perspectives*, 24(4), 45–66.

OTSU, K. (2010): "A Neoclassical Analysis of the Asian Crisis: Business Cycle Accounting for a Small Open Economy," *The B.E. Journal of Macroeconomics*, 10(1) Art. 17, 1–37.

PLOTNIKOV, D. (2017): *What prevents a real business cycle model from matching the US data? decomposing the labor wedge*. International Monetary Fund.

ROSENBERGER, A. (2013): *Krisenmanagement in der Finanz-, Wirtschafts- und europäischen Staatsschuldenkrise*, Aktuelle Fragen der Wirtschaftspolitik. Verl. für Nationalökonomie, Management und Politikberatung (NMP), Bayreuth.

SOLOW, R. M. (1957): "Technical Change and the Aggregate Production Function," *The Review of Economics and Statistics*, 39(3), 312–320.

ŠUSTEK, R. (2011): "Monetary business cycle accounting," *Review of Economic Dynamics*, 14(4), 592–612.

UHLIG, H. (1999): "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily," in *Computational Methods for the Study of Dynamic Economies*, ed. by R. Marimon, and A. Scott, 30–61. Oxford University Press, Oxford.

Appendix

4.A Model

The following equations determine the model with stationary variables

$$y_t = k_{It}^{\alpha} (Z_t N_t)^{1-\alpha}, \tag{4.A.1}$$

$$r_t = \alpha \frac{y_t}{k_{It}}, \tag{4.A.2}$$

$$w_t = (1-\alpha)\frac{y_t}{N_t}, \tag{4.A.3}$$

$$\lambda_t = \phi c_t^{\phi(1-\eta)-1} k_{Dt}^{(1-\phi)(1-\eta)} (1-N_t)^{\psi(1-\eta)}, \tag{4.A.4}$$

$$(1-\tau_{Nt}) = \frac{\psi}{\phi}\frac{c_t}{(1-N_t)w_t}, \tag{4.A.5}$$

$$y_t = c_t + i_t + d_t + g_t + e_t, \tag{4.A.6}$$

$$\mu_{It} = \lambda_t \frac{1+\tau_{It}}{1-\Theta'_{It}}, \tag{4.A.7}$$

$$\mu_{Dt} = \lambda_t \frac{1+\tau_{Dt}}{1-\Theta'_{Dt}}, \tag{4.A.8}$$

$$g_I \cdot \gamma_n k_{It+1} = (1-\delta_I)k_{It} + i_t - \Theta_{It} \cdot k_{It}, \tag{4.A.9}$$

$$g_D \cdot \gamma_n k_{Dt+1} = (1-\delta_D)k_{Dt} + d_t - \Theta_{Dt} \cdot k_{Dt}, \tag{4.A.10}$$

$$\mu_{It} = \beta g_{M_I} \mathbb{E}_t \left[\mu_{It+1} \left(1 - \delta_I - \Theta_{It+1} + \frac{i_{t+1}}{k_{It+1}} \Theta'_{It+1} \right) + \lambda_{t+1} r_{t+1} \right], \tag{4.A.11}$$

$$\mu_{Dt} = \beta g_{M_D} \mathbb{E}_t \left[\mu_{Dt+1} \left(1 - \delta_D - \Theta_{Dt+1} + \frac{d_{t+1}}{k_{Dt+1}} \Theta'_{Dt+1} \right) + \lambda_{t+1} \frac{1-\phi}{\phi} \frac{c_{t+1}}{k_{Dt+1}} \right], \tag{4.A.12}$$

with

$$g_{M_I} = g_Y^{\phi(1-\eta)} \cdot g_D^{(1-\phi)(1-\eta)} \cdot g_I^{-1}, \tag{4.A.13}$$

$$g_{M_D} = g_Y^{\phi(1-\eta)} \cdot g_D^{(1-\phi)(1-\eta)-1}, \tag{4.A.14}$$

$$\Theta_{Xt} = \frac{a_X}{2}\left(\frac{x_t}{k_{Xt}} - b_X \right)^2, \tag{4.A.15}$$

$$\Theta'_{Xt} = a_X \left(\frac{x_t}{k_{Xt}} - b_X \right), \tag{4.A.16}$$

$$b_X = x^*/k_X^*, \tag{4.A.17}$$

with $X \in \{I, D\}$, $x \in \{i, d\}$ and where * indicates the steady-state value. The fluctuation in the model is driven by the VAR(1)-process

$$\underbrace{\begin{bmatrix} \ln(s_{At+1}) \\ s_{Nt+1} \\ s_{It+1} \\ s_{Dt+1} \\ s_{Et+1} \\ \ln(s_{Gt+1}) \end{bmatrix}}_{:=\mathbf{s_{t+1}}} = \Pi \underbrace{\begin{bmatrix} \ln(s_{At}) \\ s_{Nt} \\ s_{It} \\ s_{Dt} \\ s_{Et} \\ \ln(s_{Gt}) \end{bmatrix}}_{:=\mathbf{s_t}} + \underbrace{\begin{bmatrix} \epsilon_{At+1} \\ \epsilon_{Nt+1} \\ \epsilon_{It+1} \\ \epsilon_{Dt+1} \\ \epsilon_{Et+1} \\ \epsilon_{Gt+1} \end{bmatrix}}_{:=\epsilon_{t+1}}, \qquad \epsilon_t \sim \mathcal{N}(0, \Sigma). \tag{4.A.18}$$

The stochastic process affects the wedges as follows

$$Z_t = A^* \cdot s_{At}, \tag{4.A.19}$$
$$\tau_{Nt} = \tau_N^* + s_{Nt}, \tag{4.A.20}$$
$$\tau_{It} = \tau_I^* + s_{It}, \tag{4.A.21}$$
$$\tau_{Dt} = \tau_D^* + s_{Dt}, \tag{4.A.22}$$
$$e_t = e^* + s_{Et}, \tag{4.A.23}$$
$$g_t = g^* \cdot s_{Gt}. \tag{4.A.24}$$

4.B Chari, Kehoe, and McGrattan (2007) benchmark

4.B.1 Model

$$y_t = k_t^{\alpha} (Z_t N_t)^{1-\alpha}, \tag{4.B.1}$$
$$r_t = \alpha \frac{y_t}{k_t}, \tag{4.B.2}$$
$$w_t = (1-\alpha) \frac{y_t}{N_t}, \tag{4.B.3}$$
$$\lambda_t = c_t^{(1-\eta)-1} (1-N_t)^{\psi(1-\eta)}, \tag{4.B.4}$$
$$(1-\tau_{Nt}) = \psi \frac{c_t}{(1-N_t) w_t}, \tag{4.B.5}$$
$$y_t = c_t + i_t + g_t, \tag{4.B.6}$$
$$\mu_{It} = \lambda_t \frac{1+\tau_{It}}{1-\Theta'_{It}}, \tag{4.B.7}$$
$$g_I \cdot \gamma_n k_{t+1} = (1-\delta_I) k_t + i_t - \Theta_{It} \cdot k_t, \tag{4.B.8}$$
$$\mu_{It} = \beta g_{M_I} \mathbb{E}_t \left[\mu_{It+1} \left(1 - \delta_I - \Theta_{It+1} + \frac{i_{t+1}}{k_{t+1}} \Theta'_{It+1} \right) + \lambda_{t+1} r_{t+1} \right], \tag{4.B.9}$$

with

$$g_{M_I} = g_Y^{1-\eta} \cdot g_I^{-1}, \tag{4.B.10}$$
$$\Theta_{It} = \frac{a_I}{2} \left(\frac{i_t}{k_t} - b_I \right)^2, \tag{4.B.11}$$
$$\Theta'_{It} = a_I \left(\frac{i_t}{k_t} - b_I \right), \tag{4.B.12}$$
$$b_I = i^*/k^*, \tag{4.B.13}$$

where * indicates the steady-state value.

The fluctuation in the model is driven by the VAR(1)-process

$$\underbrace{\begin{bmatrix} \ln(s_{At+1}) \\ s_{Nt+1} \\ s_{It+1} \\ \ln(s_{Gt+1}) \end{bmatrix}}_{:=\mathbf{s_{t+1}}} = \Pi \underbrace{\begin{bmatrix} \ln(s_{At}) \\ s_{Nt} \\ s_{It} \\ \ln(s_{Gt}) \end{bmatrix}}_{:=\mathbf{s_t}} + \underbrace{\begin{bmatrix} \epsilon_{At+1} \\ \epsilon_{Nt+1} \\ \epsilon_{It+1} \\ \epsilon_{Gt+1} \end{bmatrix}}_{:=\mathbf{\epsilon_{t+1}}}, \qquad \epsilon_t \sim \mathcal{N}(0, \Sigma). \tag{4.B.14}$$

The stochastic process affects the wedges as follows

$$Z_t = A^* \cdot s_{At}, \tag{4.B.15}$$
$$\tau_{Nt} = \tau_N^* + s_{Nt}, \tag{4.B.16}$$
$$\tau_{It} = \tau_I^* + s_{It}, \tag{4.B.17}$$
$$g_t = g^* \cdot s_{Gt}. \tag{4.B.18}$$

4.B.2 Observables and data manipulation

The vector of observables reads as follows $\mathbf{y_t} = \begin{bmatrix} \hat{y}_t & \hat{N}_t & \hat{i}_t & \hat{g}_t \end{bmatrix}^{\mathrm{T}}$. In contrast to our modified model government consumption is the sum of government consumption and net exports and investments are the sum of durables and investments.

4.B.3 Calibration and estimation

The calibration and estimation strategy is similar to our modified model. We estimate the elasticity of the price of capital η_I as well as the parameters of the stochastic process. All other parameters are calibrated and the long-run ratios are pined down to their long-run averages. Tables 4.B1 and 4.B2 present all relevant parameters.

Table 4.B1: Calibration and growth accounting for the Chari, Kehoe, and McGrattan (2007) economy

Parameter	description	Value
α	capital share	0.34
β	discount factor	0.994
δ_I	rate of capital depreciation	0.0203
ψ	preference weight of labor	2.24
η	risk aversion	1
η_I	elasticity of the price of capital	0.86
$\ln(\gamma_n^4)$	annual growth rate of population	0.03%
$\ln(g_Y^4)$	annual growth rate of GDP	1.32%
$\ln(g_I^4)$	annual growth rate of investment	0.79%

Table 4.B2: Estimation of exogenous shock process of the Chari, Kehoe, and McGrattan (2007) economy

	Autoregressive Matrix				
Π	$ln(s_A)$	s_N	s_I	$ln(s_G)$	
$ln(s_A)$	0.93	0.09	0.05	−0.03	
s_N	−0.01	0.73	0.04	−0.00	
s_I	0.03	2.03	0.67	−0.02	
$ln(s_G)$	0.09	−1.17	0.08	0.84	
	Correlation and standard errors				
$Corr(\epsilon_i, \epsilon_j)$	ϵ_A	ϵ_N	ϵ_I	ϵ_G	$100 \cdot StD(\epsilon_i)$
ϵ_A	1.00				0.94
ϵ_N	0.21	1.00			0.29
ϵ_I	−0.27	−0.61	1.00		1.77
ϵ_G	0.43	0.77	−0.34	1.00	2.71

4.C Data

The data is taken from the Fachserie 18: National accounts, domestic product from the German Federal Statistical Office.

- **Pop:** Total Population 1991:I-2018:I

  Source: 2.1.7 Population and labour force participation 1; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **Hours:** Hours worked by persons in employment 1991:I-2018:I

  Source: 2.1.8 Persons in employment, employees and hours worked (domestic concept) 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **GDP:** 1991:I-2018:I

  Nominal source: 2.3.1 Use of gross domestic product at current prices 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

  Real source: 2.3.2 Use of gross domestic product, price-adjusted 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **PCE:** Private Consumption Expenditures of households 1991:I-2018:I

  Nominal source: 2.3.3 Final consumption expenditure at current prices 3; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

  Real source: 2.3.4 Final consumption expenditure at , price-adjusted; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **Govern. Consumption:** Government final consumption expenditure (domestic use) 1991:I-2018:I

Nominal source: 2.3.3 Final consumption expenditure at current prices 3; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

Real source: 2.3.4 Final consumption expenditure at , price-adjusted; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **Investment:** Gross fixed capital formation 1991:I-2018:I

  Nominal source: 2.3.1 gross fixed capital formation at current prices 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

  Real source: 2.3.2 gross fixed capital formation, price-adjusted 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **Net Exports:** Balance of exports and imports 1991:I-2018:I

  Nominal source: 2.3.1 Balance of exports and imports at current prices 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

  Real source: 2.3.2 Balance of exports and imports, price-adjusted 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **Durables:** Langlebige Güter (Durable Goods) 1991:I-2018:I

  Nominal source: 2.14 Konsumausgaben der privaten Haushalte im Inland nach Dauerhaftigkeit der Güter, Saison- und kalenderbereinigt in jeweiligen Preisen 4; Private Konsumausgaben und Verfügbares Einkommen - 1. Vierteljahr 2018

  Real source: 2.14 Konsumausgaben der privaten Haushalte im Inland nach Dauerhaftigkeit der Güter, Saison- und kalenderbereinigt - preisbereinigt 4; Private Konsumausgaben und Verfügbares Einkommen - 1. Vierteljahr 2018

  (available in German only: Domestic consumer spending on durable goods, seasonally and calendar adjusted 4; Private consumption expenditure and disposable income - 1st quarter of 2018)

Chapter 5

Conclusion

All three essays in this thesis deal with heterogeneous assets in Dynamic Stochastic General Equilibrium (DSGE) models by differentiating between productive market and utility augmenting private assets. In Chapter 2 and 3 the utility augmenting asset represents housing whereas in Chapter 4 durable consumption goods.

All models in Chapter 2 and 3 try to account for common puzzling stylized facts from the housing and the business cycle literature inside the Real Business Cycle (RBC) framework, where markets are perfect and fluctuations are induced by the supply side. These stylized facts read as follows: i) residential investment is at least moderately more volatile than business investment, ii) house prices are at least twice as volatile as Gross Domestic Product (GDP), and iii) house prices, business investment as well as GDP are positively correlated with residential investment.

In Fehrle (2019) (Chapter 2), I show that the multi-sectoral model from Davis and Heathcote (2005), which is a baseline in the RBC literature concerning housing, accounts for correlated movements between residential investment and business investment only if shocks towards sectoral productivity are jointly distributed. The explanatory power of the model generally worsens without correlated shocks. However, I show that by introducing adjustment costs and variable capital utilization the model's explanatory power without correlated shocks becomes at least as good as the baseline of Davis and Heathcote (2005) with correlated shocks.

In Fehrle and Heiberger (2020) (Chapter 3), we additionally take stylized facts of asset return statistics into account. These are i) a stable risk-free rate smaller than 2.25 percent, ii) return rates on equity moderately larger than returns on housing, iii) risk premia on equity, on housing and on total risk larger than 3 percent, iv) return rates and premia on equity which are at least twice as volatile as return rates and premia on housing and on total risk, and v) a Sharpe ratio of housing significantly larger than the Sharpe ratio of equity. In models which rely on habit formations and capital adjustment costs, housing provides an insurance against fluctuations in marginal utility as long as the transformability between new houses and consumption is sufficiently feasible. The insurance induces a demand effect which contributes to explanations of puzzling business cycle statistics. However, since the household has an insurance option, he charges too low risk premia to take over aggregated risk. As the transformability becomes insufficient, e.g. by introducing limited sectoral mobility, the insurance disappears, and certainly, the household charges high risk premia. However, the demand effect disappears as well, and as a consequence, the model does not contribute to explanations of puzzling business cycle characteristics. An otherwise standard RBC model with housing and time-varying disaster risk is more promising. Since the household cannot insure against rare disasters, he charges sizable risk premia. Further, the model explains the different risk premia of housing and equity, the volatility of the return on aggregated risk, housing and the risk-free rate as well as for the stylized facts concerning the business cycle statistics. Time-varying risk contributes heavily to explanations of the mentioned stylized facts. The major disadvantage of the model is its inability to disentangle the Sharp ratios of equity and housing.

In Fehrle and Huber (2020) (Chapter 4), we apply the Business Cycle Accounting (BCA) procedure proposed by Chari, Kehoe, and McGrattan (2007) to investigate the Great Recession in Germany and the subsequent stimulus measures. Since the literature reports that the procedure is somewhat difficult and there is no straight methodological implementation of BCA, we first propose a well-performing procedure. This procedure includes approaches we found scattered in the literature or we developed by ourselves. Afterwards, we quantify the contribution to the business cycle of distortions in the new durable consumption goods, the productive investment goods, and the labor market, as well as the contribution of changes in efficiency, government consumption, and net exports. Our results suggest that the recession was mainly driven by disruptions in efficiency. Distortions in the market for business investments and a decrease in net exports also contributed substantially to the crisis. Higher government spending and in particular distortions in the market for new durable consumption goods acted counter-cyclically. We attribute both counter-cyclical effects to the fiscal stimulus and conclude that the measures in the durable markets, namely durable subsidies, were more efficient. The different signs in the contribution to the business cycle of new durables and productive investments indicate that their segregation is a necessity for the object of the present investigation. Thus, previous studies applying BCA for the Great Recession in Germany underrate distortions of investments in productive assets by treating them as an aggregator including new durables. Lastly, our results suggest that the recovery was driven by a fast rehabilitating labor market. However, the assignment of the labor market drivers' determinants is not as apparent as of other determinants, e.g., of the durable market distortions.

Bibliography

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2007): "Business Cycle Accounting," *Econometrica*, 75(3), 781–836.

Davis, M. A., and J. Heathcote (2005): "Housing and the business cycle," *International Economic Review*, 46(3), 751–784.

Fehrle, D. (2019): "Housing and the business cycle revisited," *Journal of Economic Dynamics and Control*, 99, 103–115.

Fehrle, D., and C. Heiberger (2020): "The return on everything and the business cycle in production economies".

Fehrle, D., and J. Huber (2020): "Business cycle accounting for the German fiscal stimulus program during the Great Recession".